
Patent Value and Citations: Creative Destruction or Strategic Disruption?

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Patent Statistics for Decision Makers

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Introduction

Introduction

- **Value of innovation is a crucial input for**
 - Innovation studies
 - Industrial Organization
 - Economic Growth Theory
 - **Critical Policy Decisions...**

Introduction

...such as

- **How to promote innovation?**
- **What type of innovation to promote?**
- **Do entrepreneurs produce the most valuable innovation?**
- **Are NPE's good or bad for innovation?**

Introduction

- **What proxies are used?**
 - Patent count
 - Intuition: more valuable innovation → more patents
- **But...patents vary enormously in value**
 - Fat tailed distribution
 - From patent renewal studies (e.g. Pakes 1986; Schankerman & Pakes 1986; Bessen 2008)
 - Only 10% worth the cost (Allison, Lemley, Moore, Trunkey 2009)

Introduction

- **Use citation-weighted patent counts**
 - Intuition: more valuable patents receive more subsequent citations (forward citations)
- **Many papers have relied on this measure, e.g.**
 - Lerner and Kortum (2000)
 - Jaffe, Trajtenberg, Romer (2002)
 - Aghion, Bloom, Blundell, Griffith, Howitt (2005)
 - Abrams (2009)

Introduction

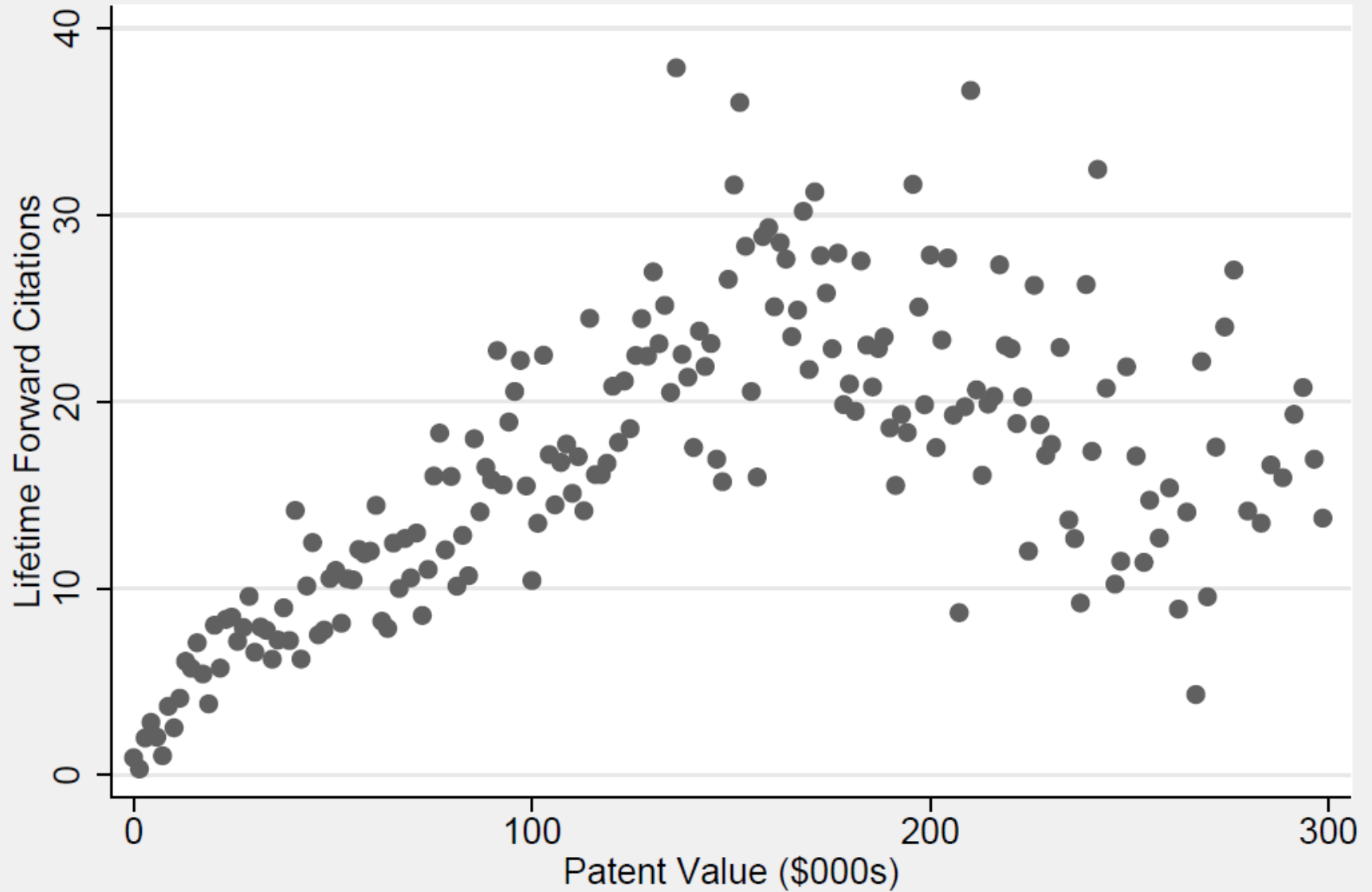
Big literature uses citations, but few papers investigate its validity:

- **Trajtenberg (1990)**
 - **Individual patent specific social value for Computed Tomography Scanners.**
- **Hall, Jaffe and Trajtenberg (2005)**
 - **Stock market value**
- **Harhoff, Scherer and Vopel (1999, 2003), Gambardella, Harhoff and Verspagen (2005)**
 - **Survey of inventors.**
- **Bessen (2008)**
 - **Patent renewals.**

Introduction

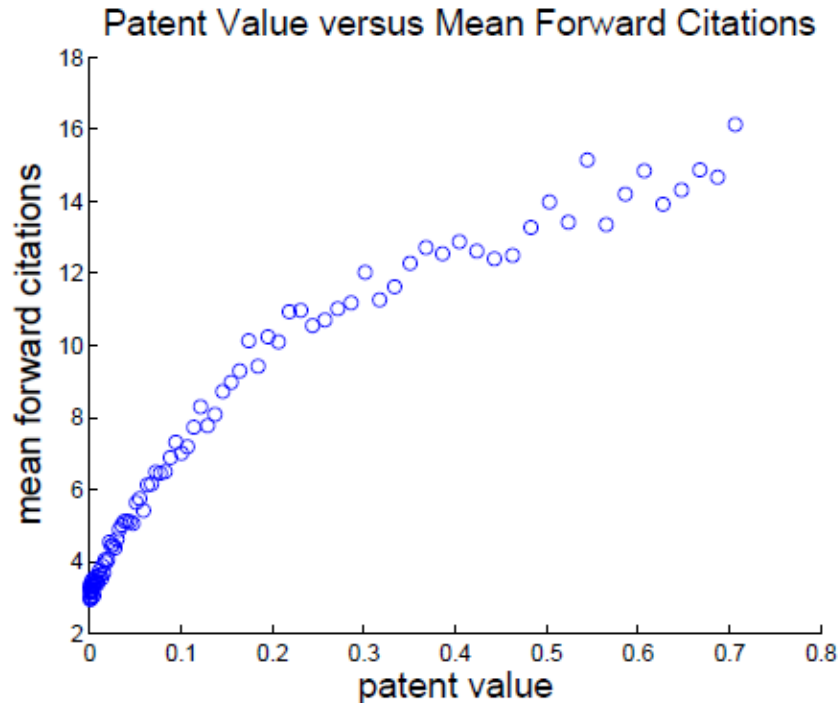
- **Today**
 - Explore the citation-value relationship
 - Learn about NPE's
- **First Data Available with:**
 - Large N: tens of thousands of patents from NPE's
 - Many Technology Classes (248 USPTO class codes)... and
 - **Actual Patent-Specific Revenues**

Forward Citations vs. Patent Value



Introduction

- **What can explain this finding?**
 - Standard theory of creative destruction predicts

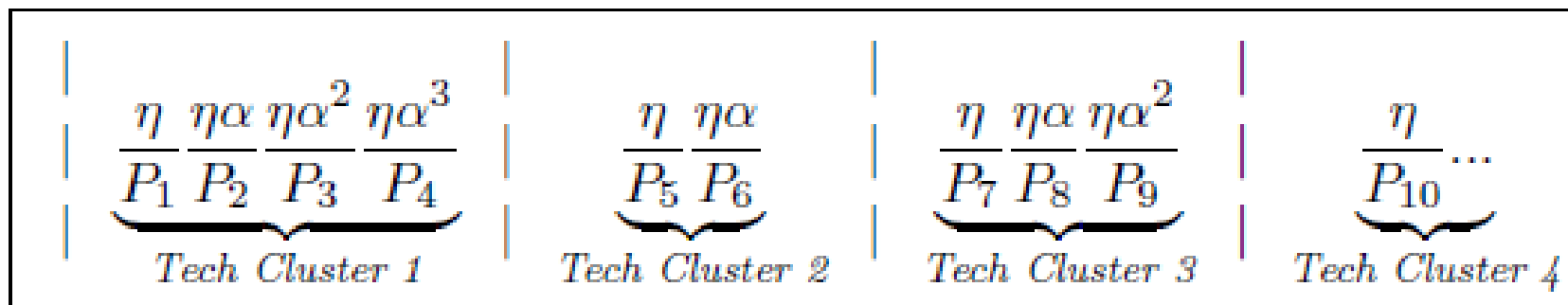


Introduction

- **We propose a new theory with**
 - Productive innovations
 - Strategic innovations
- **Model accounts for inverted-U**
- **Produces other testable predictions**

Model of Innovation

Example for Productive Innovations



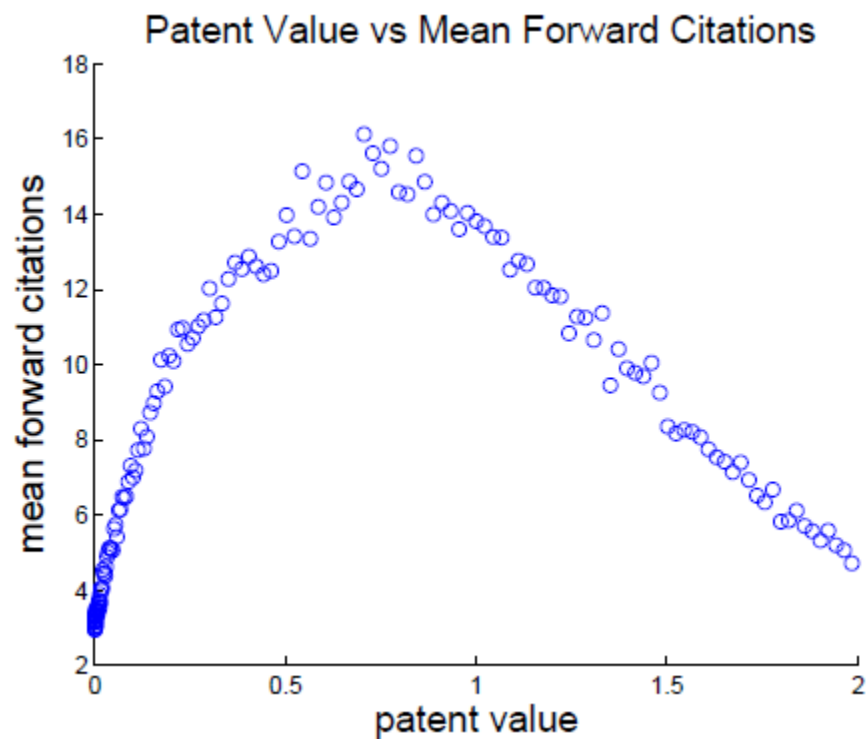
AN EXAMPLE OF A SEQUENCE OF INNOVATIONS IN A PRODUCT LINE

<i>Cited</i>	<i>Citing</i>	<i>Cited</i>	<i>Citing</i>
P_1	P_2, P_3, P_4	P_6	<i>none</i>
P_2	P_3, P_4	P_7	P_8, P_9
P_3	P_4	P_8	P_9
P_4	<i>none</i>	P_9	<i>none</i>
P_5	P_6	P_{10}	\dots

Model Summary

- Radical productive patents generate high market value and attract subsequent entry through spillovers.
 - Initial positive link between value and citations
- Above a certain value threshold, incumbents find it worthwhile to pay the fixed cost and produce strategic patents to prevent entry.
 - High value implies less subsequent entry and fewer citations, i.e., a negative relationship.
- Overall, an **inverted-U relationship** between patent value and citations.

Productive and Strategic Innovations together



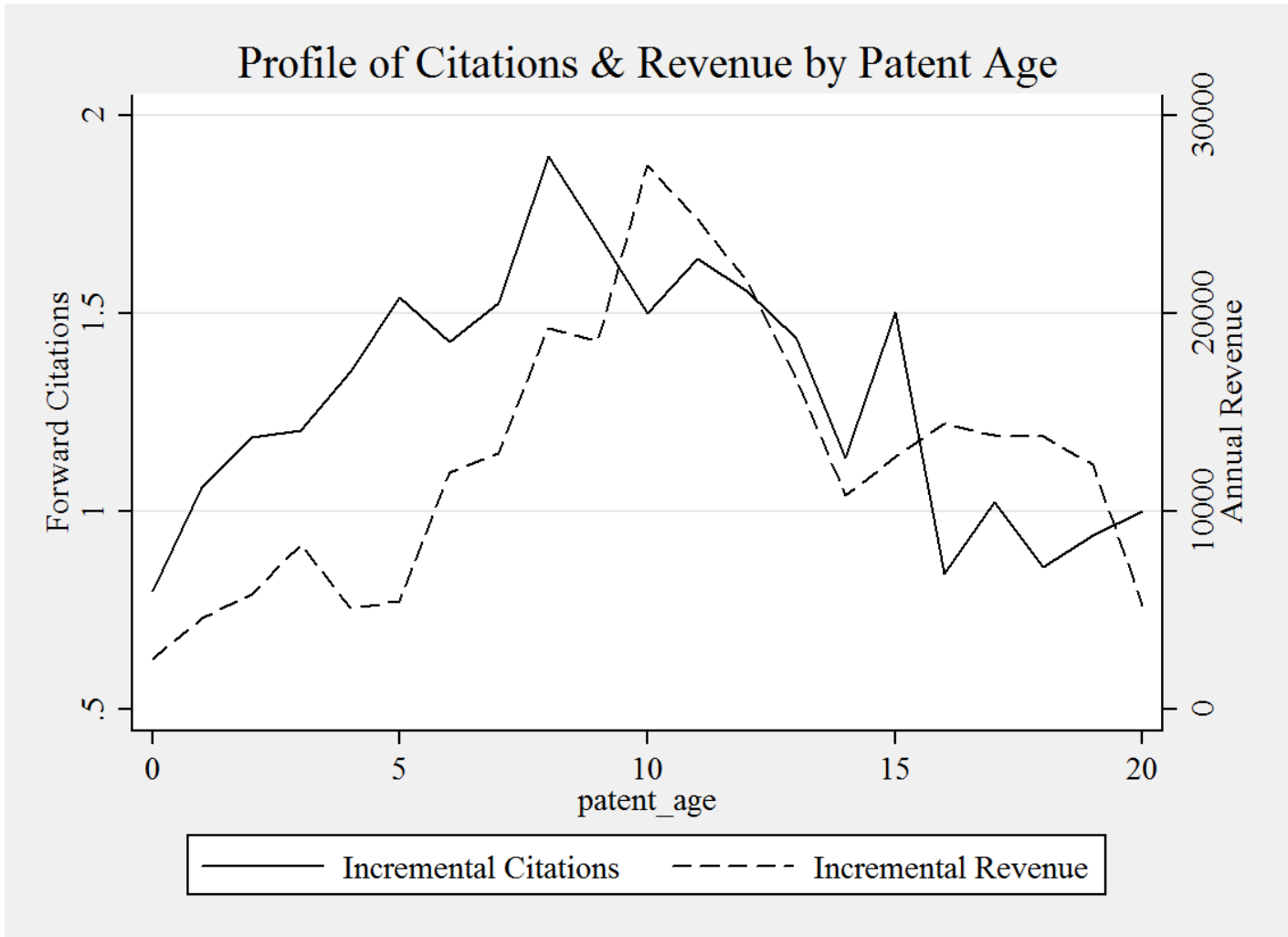
Data

Revenue Allocation

- **Confidentiality agreements put some limits on what we can disclose.**
- **We cannot identify the data sources, nor the exact level of revenues.**
- **But we can report a lot of information about the data set:**
 - **Tens of thousands of patents**
 - **Patent-year-licensee level revenues between 2008-2012 which we aggregate to the patent-year level**

Revenue and Licensing Deals

- **Almost all revenue is derived from licensing patents to customers**
- **Patents are usually licensed in portfolios of hundreds or thousands**
- **Each patent is generally licensed to multiple parties**
- **The prominence of a patent in a licensing deal impacts its' revenue allocation**
- **Multiple parties have strong financial incentives for revenue allocations to be accurate**



Data

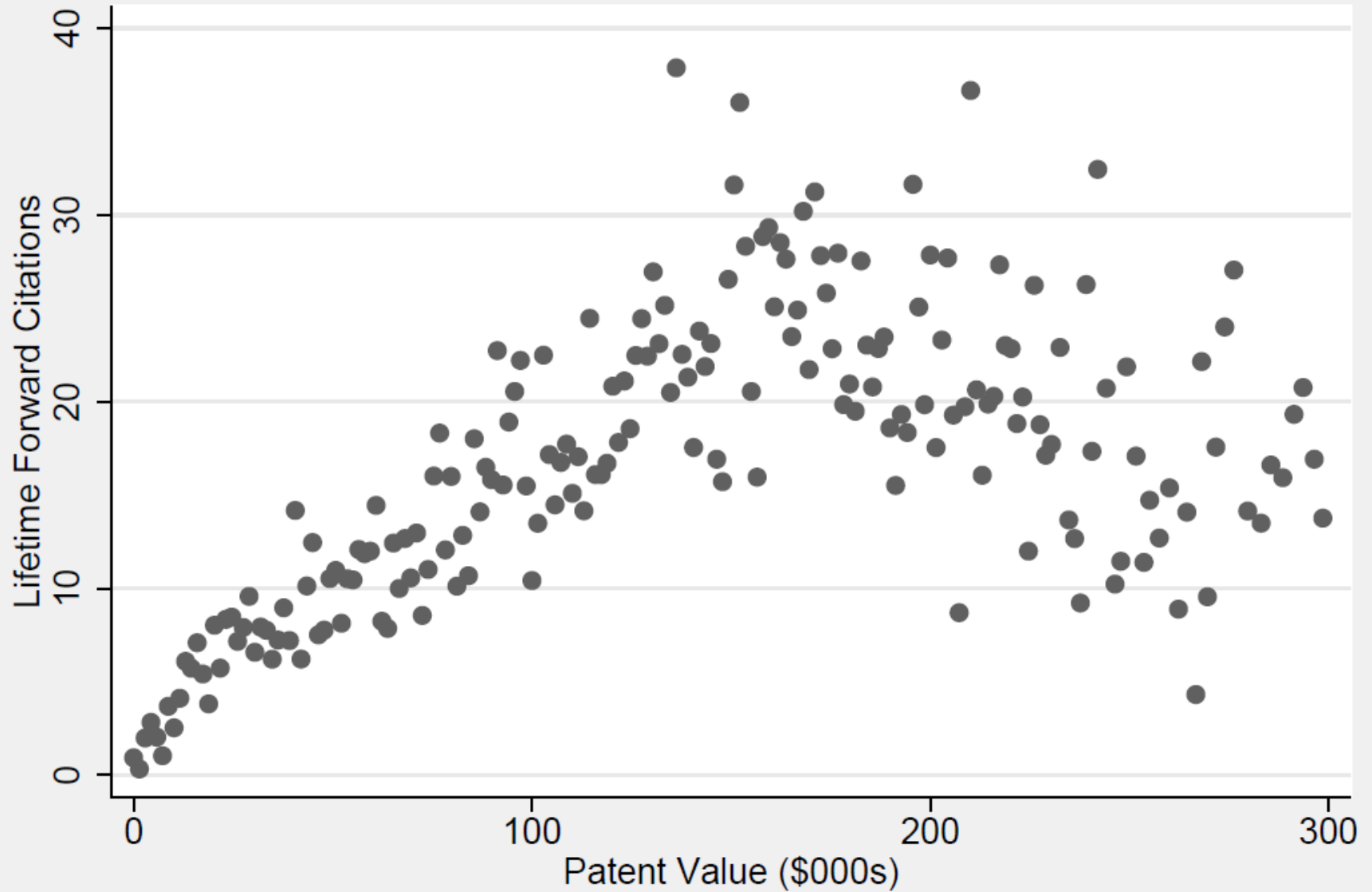
Patent-year-licensee level observations

	Mean	Standard Deviation	Median
Patent Value (\$000s)	204.2	1904.7	52.19
Lifetime Forward Citations	29.1	52.5	11.5
Backward Citations	23.1	59.9	8.0
Fraction of Backward Cites in Past 3 Years	0.20	0.30	0.00
Fraction of Backward Cites in Past 5 Years	0.28	0.37	0.00
Original Indicator	0.84	0.36	1.00
Application Year	1999	4.7	1999
Individual Inventor Indicator	0.14	0.35	1.00

Note: Data is normalized so that the mean annual revenue is \$10,000 (2010\$). Original patent applications are those which are not divisionals or continuations.

Analysis

Forward Citations vs. Patent Value



Forward Citations vs. Patent Value

	Share of most valuable patents excluded					
		10%		5%		1%
Patent Value (\$100,000s)	9.047** (0.256)	22.497** (0.654)	7.104** (0.232)	14.402** (0.566)	6.961** (0.246)	8.016** (0.432)
Patent Value Squared		-6.036** (0.288)		-2.193** (0.195)		-0.139* (0.070)
R^2	0.04	0.05	0.04	0.05	0.09	0.09

** Significant at the 1% level; * Significant at the 5% level

Note: Separate regressions reported in each column, with standard errors in parentheses. Dependent variable is lifetime forward citations. Data is normalized so that the mean annual revenue is \$10,000 (2010\$). Regression excludes indicated top percent of patents by value.

Determinants of Forward Citations

	(1)	(2)	(3)	(4)
Patent Value (\$100,000s)	7.569** (0.622)	9.272** (0.637)	8.669** (0.631)	8.444** (0.615)
Patent Value Squared	-0.906** (0.205)	-1.254** (0.206)	-1.213** (0.206)	-1.130** (0.201)
Individual Inventor	-18.512** (0.388)	-18.364** (0.385)	-17.141** (0.406)	-17.209** (0.399)
Patent Application Before 2000		5.347** (0.332)	5.968** (0.330)	6.337** (0.332)
Indicator Original Patent			-7.583** (0.682)	-5.384** (0.659)
Tech Category (Computer Architecture)				3.632** (0.565)
Tech Category (Electro-Mechanical)				4.03** (0.642)
Tech Category (Internet & Software)				19.87** (0.872)
Tech Category (MEMS & Nano)				3.798** (1.314)
Tech Category (Networking & Communications)				9.808** (0.734)
Tech Category (Optical Networking)				2.1** (0.472)
Tech Category (Peripheral Devices)				2.508** (0.413)
Tech Category (Semiconductors)				3.387** (0.431)
Tech Category (Wireless Communications)				7.22** (0.524)
R^2	0.12	0.12	0.13	0.16

** Significant at the 1% level; * Significant at the 5% level

Note: Separate regressions reported in each column, standard errors in parentheses. Dependent variable is lifetime forward citations; circuits is the excluded technology category. Data is normalized so that the mean annual revenue is \$10,000 (2010\$).

**The inverted-U supports the theory of
productive and strategic patenting.**

But further evidence is needed.
We test 4 predictions of the theory.

Prediction #1

- **Theory**: The cost to attempt a strategic innovation is more easily borne by larger entities
- **Prediction**: Large-entities are more likely to employ strategic patenting than individuals and small-entities

Forward Citations vs. Patent Value By Share of Corporate Assignees

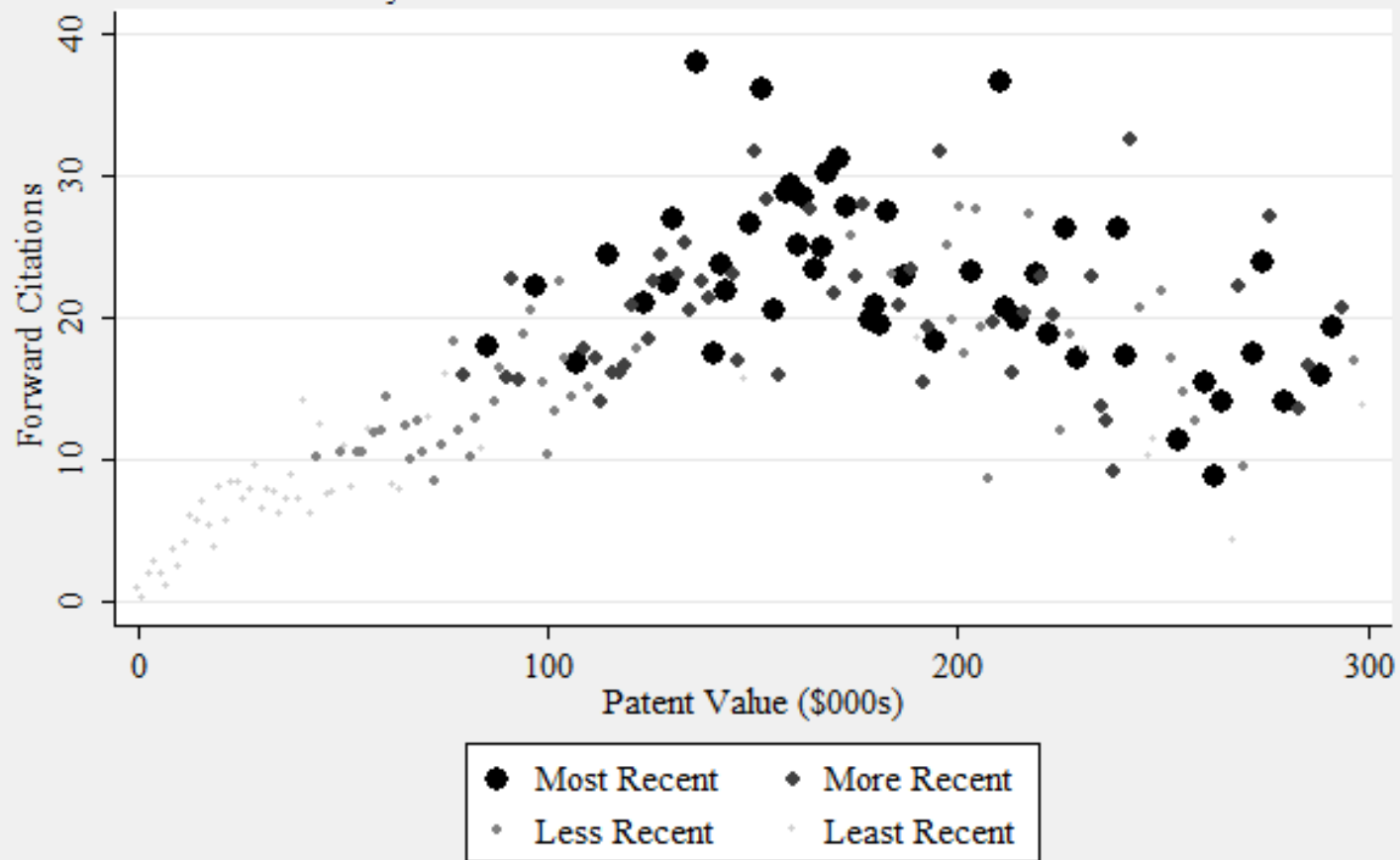


Prediction #2

- Theory: Greater profits are available in fields of rapid growth.
- Prediction: Strategic patenting will be more common when backward citations are concentrated in recent years.

Forward Citations vs. Patent Value

By Recent Concentration of Backward Citations

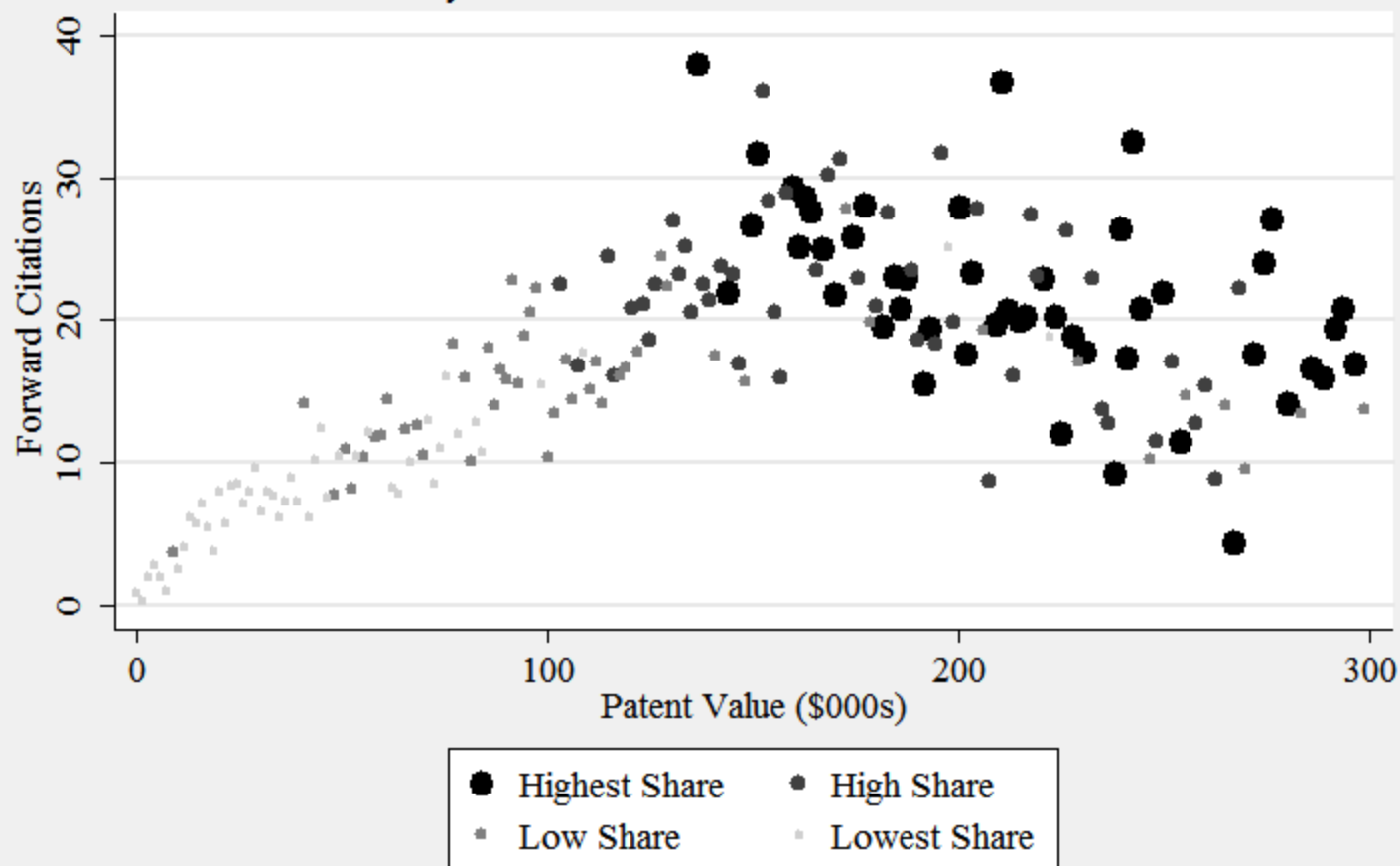


Prediction #3

- Theory: More sophisticated and costly patenting strategies should be more prevalent for strategic innovations.
- Prediction: Divisional and Continuation patents will be more commonly used for strategic purposes.

Forward Citations vs. Patent Value

By Share of Continuations and Divisionals

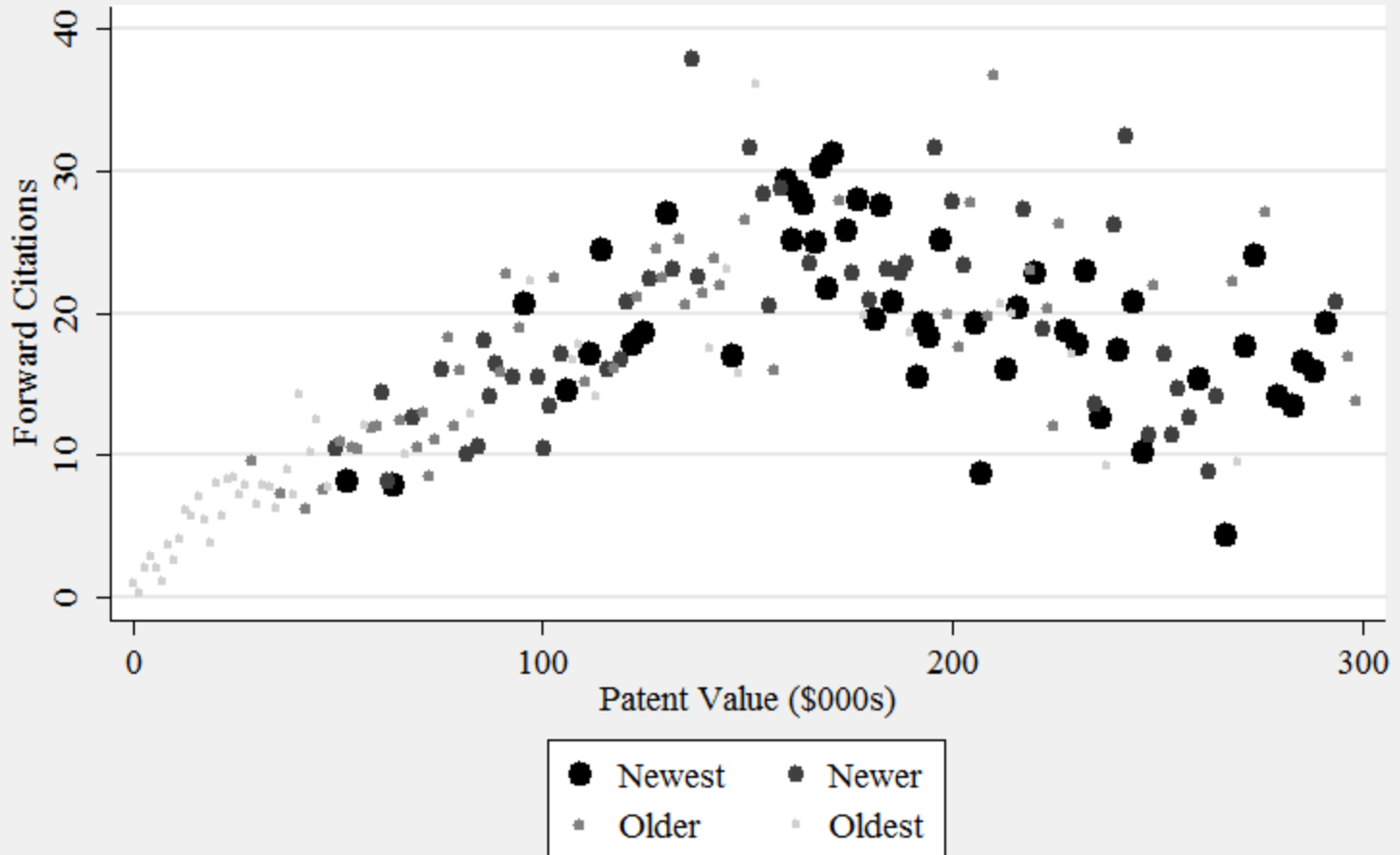


Prediction #4

- Theory: Strategic innovation is increasing over time perhaps due to higher returns
- Prediction: Newer patents will comprise a larger share of strategic patents.

Forward Citations vs. Patent Value

By Patent Age



**All four tests are consistent with
productive and strategic patenting.**

Conclusion

- **We build on the prior work on patent value and citations and confirm that the correlation is positive. But our data indicates that the relationship is more complex.**
- **The citation-value relationship has an inverted-U shape**
- **Our model and data provide strong evidence for the strategic use of patents, a topic of substantial recent interest.**
- **While our results may not generalize to all USPTO patents, our sample's extensive coverage of technology patents should help illuminate major policy discussions.**

End

**Does the relationship hold within a
technology class?**

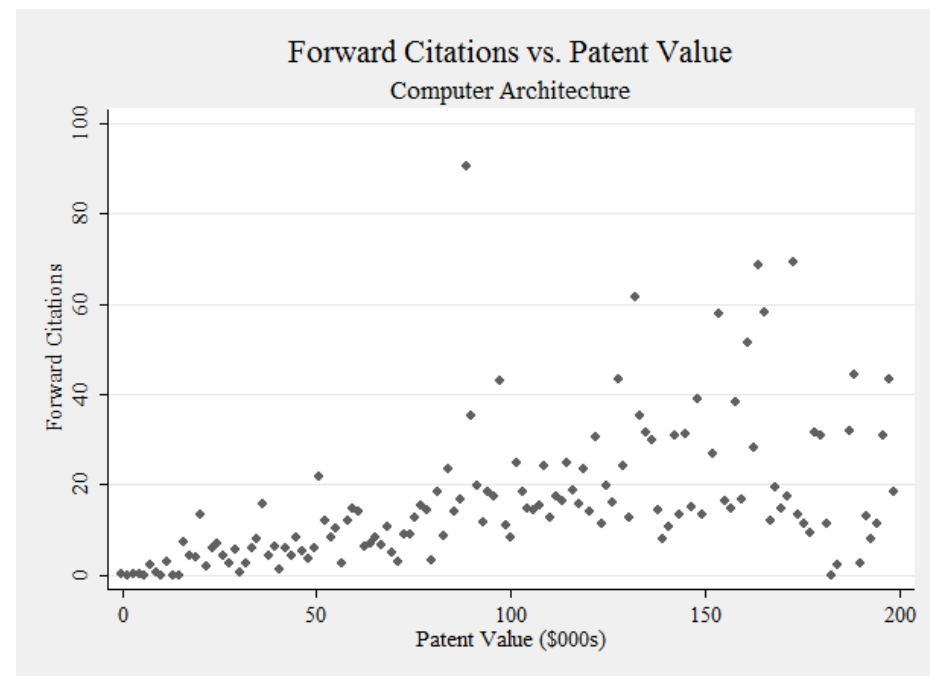
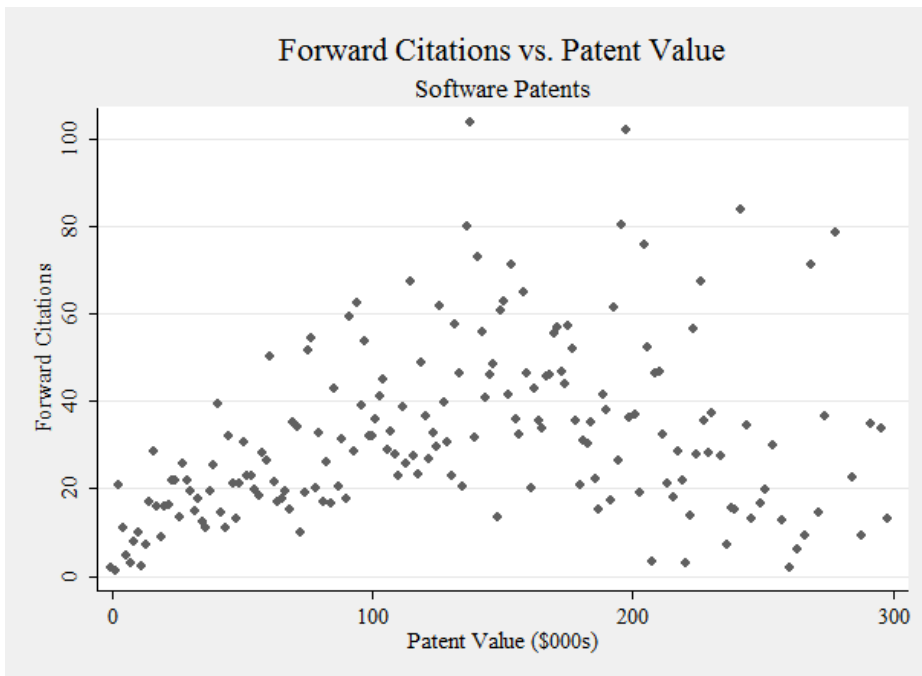
Patent Value and Cites by Technology

Technology	Patent Value	Lifetime Forward Citations
Circuits	\$367,130	7.1
Computer Architecture	\$283,773	6.0
Internet & Software	\$273,093	12.6
Wireless Communications	\$174,605	35.4
Network Communications	\$146,974	9.4
Semiconductor Devices	\$115,824	7.8
Peripheral Devices	\$99,801	8.1
Electro-Mechanical	\$62,018	7.4
MEMS & Nano	\$58,860	11.1
Optical Networking	\$56,425	16.5

Note: Data is normalized so that the mean annual revenue is \$10,000 (2010\$).

Results by Technology Category

- The Inverted-U holds across technology categories



Inverted-U Robust Across Technologies

	Circuits	Computer Architecture	Electro-Mechanical	Internet & Software	MEMS & Nano
Patent Value (\$100,000s)	6.233 (6.89)**	14.497 (11.28)**	10.917 (6.60)**	23.542 (10.95)**	17.051 (4.75)**
Patent Value Squared	-0.777 (3.18)**	-2.212 (6.27)**	-2.341 (3.93)**	-3.184 (4.39)**	-4.325 (3.80)**
R^2	0.05	0.09	0.04	0.05	0.06

	Networking Communication	Optical Networking	Peripheral Devices	Semiconductors	Wireless Communications
Patent Value (\$100,000s)	19.107 (8.64)**	13.496 (11.43)**	9.847 (14.64)**	9.329 (9.60)**	18.007 (12.04)**
Patent Value Squared	-2.328 (2.90)**	-2.114 (4.57)**	-2.355 (11.09)**	-1.020 (3.01)**	-3.292 (5.91)**
R^2	0.08	0.07	0.02	0.06	0.07

** Significant at the 1% level; * Significant at the 5% level

Note: Separate regressions reported in each column, t-statistics in parentheses. Dependent variable is lifetime forward citations. Data is normalized so that the mean annual revenue is \$10,000 (2010\$).

**How does the inventor type correlate
with patent characteristics?**

Data

Type of innovator is Extremely Important

Summary Statistics by Inventor Type

	Individual Inventor	Private Company	Public Company
Lifetime Revenue (\$000s)	81.8	242.2	270.8
Lifetime Forward Citations	3.7	26.8	33.7
Backward Citations	4.2	24.3	21.6
Concentration of Backward Cites in Past 3 Years	37%	46%	49%
Concentration of Backward Cites in Past 5 Years	56%	64%	67%
Original Indicator	93%	67%	74%
Application Year	1999	2001	1998

Note: Data is normalized so that the mean annual revenue is \$10,000. Original patent applications are those which are not divisionals or continuations.

**How do these findings compare with
other research on patent value?**

Mean Lifetime Revenue and Citations by Technology

Technology	Mean Revenue	Median Revenue	Mean-to-median Revenue	Mean Citations	Median Citations	Mean-to-Median Citations
Internet & Software	273,093	29,449	9.3	21.4	17.3	1.2
Wireless Communications	174,605	20,631	8.5	7.9	7.3	1.1
Circuits	367,130	48,316	7.6	6.0	5.3	1.1
Network Communications	146,974	21,670	6.8	16.6	11.0	1.5
Computer Architecture	283,773	43,133	6.6	16.3	7.6	2.1
Peripheral Devices	99,801	17,813	5.6	4.2	4.1	1.0
Semiconductor Devices	115,824	21,269	5.4	9.8	6.4	1.5
Electro-Mechanical	62,018	18,305	3.4	9.6	6.2	1.5
Optical Networking	56,425	32,231	1.8	5.9	4.3	1.4
MEMS & Nano	58,860	33,693	1.7	7.1	3.9	1.8
Total	177,743	23,554	7.5	11.2	6.2	1.8

*Normalized such that the mean annual revenue per patent is \$10,000.

Other Estimates of Patent Value

Renewal Method

Bessen (2009)

- U.S. Patents
- Mean Value \$121,472
- Median Value \$11,148

Equity Method

Hall, Jaffe, and Trajtenberg (2005)

- U.S. Corporate Patents
- Mean Value \$1,000,000

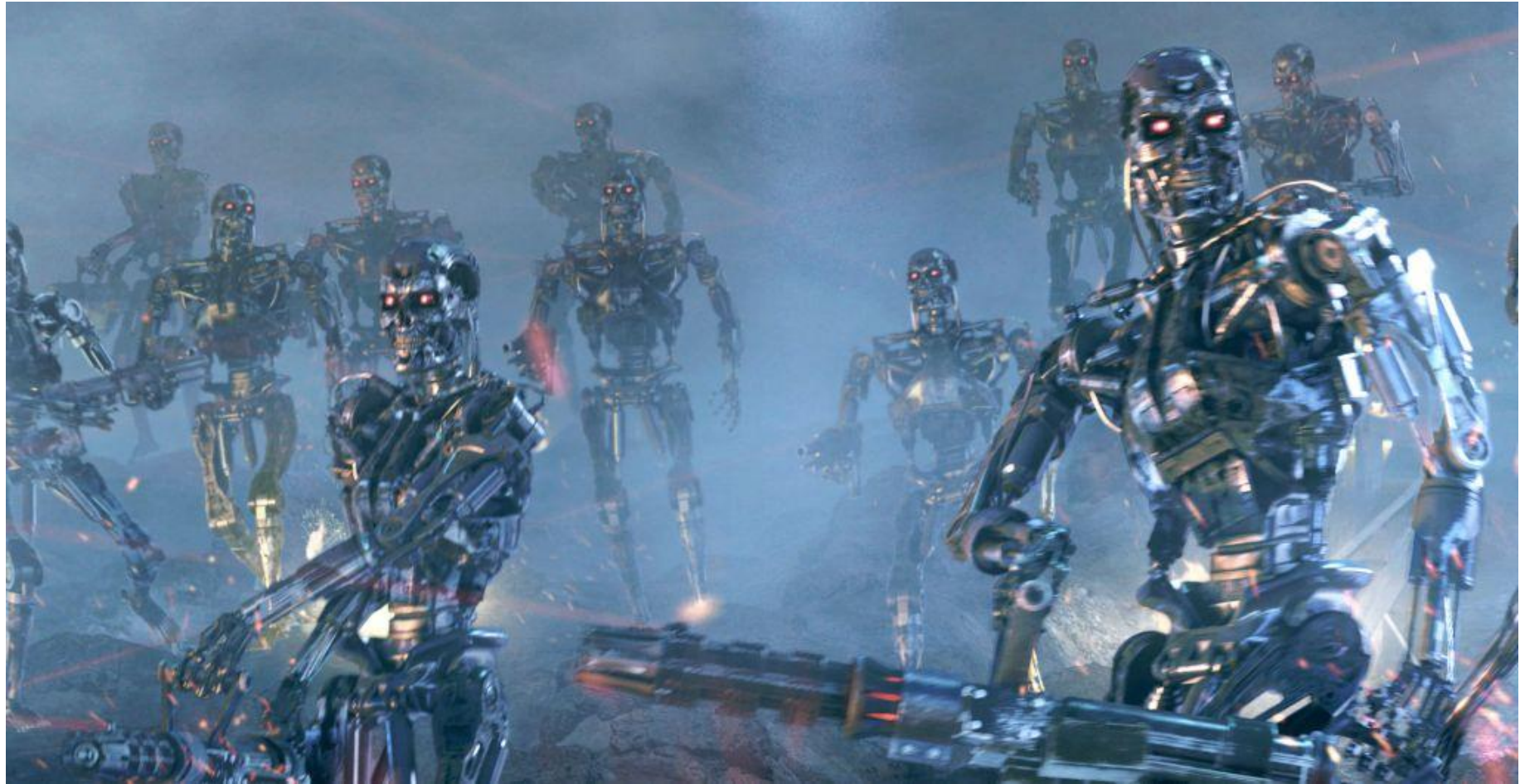
Transfer Method

Serrano (2010)

- U.S. Patents
- Mean Value \$90,799
- Median Value \$19,184

Technology	Mean	Median	Mean-to-Median
Chemical	772,650	52,612	14.7
Mechanical	133,695	12,698	10.5
Drugs & Medical	187,131	19,723	9.5
Other	60,025	7,106	8.4
Electrical & Electronic	106,385	18,536	5.7
Computers & Communication	70,314	33,080	2.1

*All values are in \$2010 dollars; table pulled from Bessen (2009).



Rise of the Machines Learning

What Is Machine Learning

- **Machine Learning is the extraction of implicit, previously unknown, and potentially useful information from data.**
- **Finding strong patterns help to make accurate predictions on future data.**
- **The goal is to find an algorithm robust enough to cope with imperfect data and imprecise patterns.**

Other Patent Characteristics May Affect Value

- **Who made the citation (examiner, self, competitor, etc...)?**
 - Hall, Jaffe, Trajtenberg (2005); Hedge & Sampat (2009)
- **Family Size / International Protection**
 - Harhoff, Schere, & Vopel (2003); Lanjouw & Schankerman (2004)
- **Details of Invention (Scope, Depth, Dependent Claims)**
 - Lerner (1994); Moser, Ohmstedt, & Rhode (2012)
- **Inventors, All-star inventor**
 - Zucker et al. (2002)
- **Assignee Type (Public, Govt, Private Firm, Individual Inventor)**
 - Thursby & Thursby (2005); Arora et al. (2008); Bessen (2008)

Industry Characteristics May Affect Patent Value

- **Competition limits rents extracted**
 - Blundell et al. (1999)
 - Aghion et al. (2005)
- **Market maturity, growth opportunities**
 - Hopenhayn, Llobet, and Mitchell (2006)
- **Consumers willingness-to-pay for innovation**
 - Weyl and Tirole (2013)
- **Other: spillover, ownership concentration, etc...**
 - Bloom, Schankerman, Van Reenen (2013)
 - Aghion, Stein, Zingales (2013)

Firm & CEO Characteristics May Affect Value

- **Financing Constraints**
- **Asymmetric Information in the Market**
- **CEO Career Concerns**
 - Younger, early tenure CEOs invest in low variance patents Holmstrom (1982), Manso (2011)
 - Optimistic/overconfident CEOs overinvest in high variance patents (Malmendier and Tate 2005; Ben-David, Graham, Harvey 2013)

How Do We Make Sense of So Many Factors

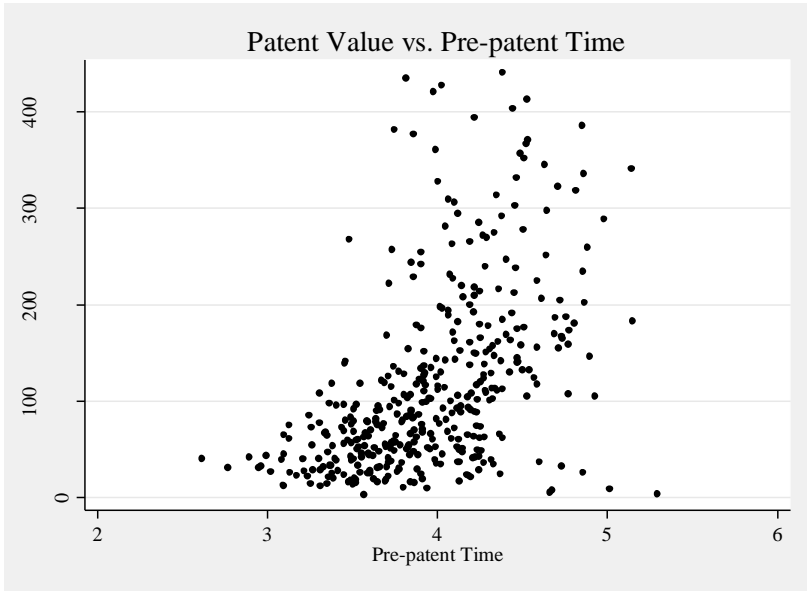
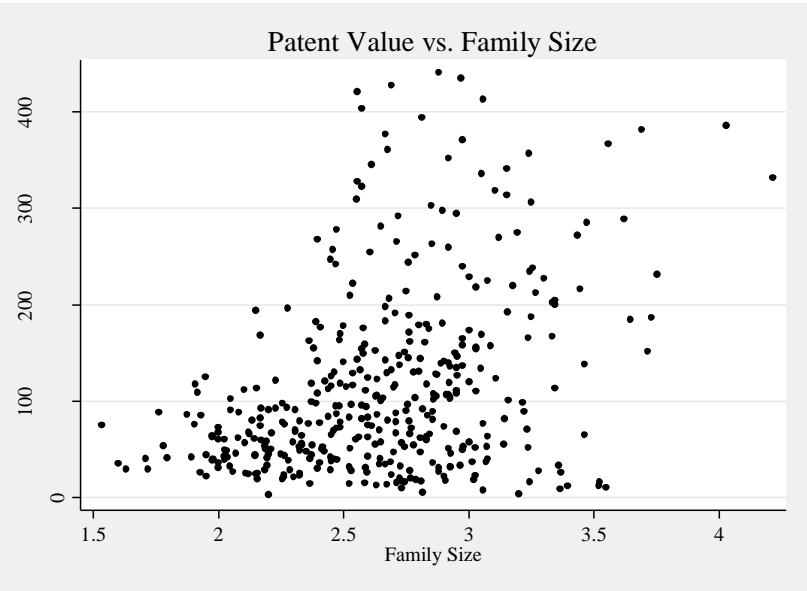
- **Evaluate covariates**
 - Correlation, Linear, and Quadratic relations
- **Variable Selection**
 - LASSO, Ridge and Bayesian techniques
- **Apply machine learning techniques to create improved patent value proxy by allowing for interactions, non-linearities, and blended models.**

U.S. Patent Characteristics

	Mean	Standard Deviation
Citations	25.4	44.4
Backward Cites	21.3	58.9
Recent Tech	64.0%	28.2%
Claims	19.8	15.4
Dependent Claims	16.4	14.0
Prepatent Time	4.6	2.7
Breadth	1.6	0.8
Indepeth	3.2	2.3
Inventors	2.1	1.5
Allstar Inventor	9.0%	29.5%
Reissuance	1.4%	11.7%
International Assignee	46.2%	49.9%
Original	71.4%	45.1%
Individual Inventor	14.5%	35.2%
Public Firm	46.8%	49.9%
Private Firm	29.3%	45.5%

Note: Data is normalized so that the mean annual revenue is \$10,000 (2010\$). Original patent applications are those which are not divisionals or continuations.

Other Patent Characteristics



Variable Selection

	Linear	LASSO Linear
Forward Citations	0.05 (0.008)***	0.05 (0.008)***
Backward Citations	0.037 (0.009)***	0.037 (0.009)***
Recent Technology	0.038 (0.009)***	0.038 (0.009)***
Breadth	0.043 (0.009)***	0.043 (0.009)***
Claims	0.078 (0.046)*	0.056 (0.008)***
Dependent Claims	-0.022 (0.046)	
Family Size	-0.013 (0.009)	-0.013 (0.009)
Inventors	-0.007 (0.008)	-0.007 (0.008)
Prepatent-Time	0.018 (0.013)	0.018 (0.013)
Indepth	0.004 (0.009)	0.004 (0.009)
All-star Inventors	0.144 (0.028)***	0.144 (0.028)***
Reissuances	0.182 (0.064)***	0.182 (0.064)***
International Assignees	0.121 (0.018)***	0.12 (0.018)***
Original	-0.206 (0.021)***	-0.205 (0.021)***
Year Fixed Effects	Yes	Yes
Technology Fixed Effects	Yes	Yes
USPTO Fixed Effects	Yes	Yes
Examiner Fixed Effects	Yes	Yes
R^2	37%	37%

* p<0.1; ** p<0.05; *** p<0.01

- Variable Selection technique is LASSO: Least Absolute Shrinkage and Selection Operator.
- Penalizes covariates that are redundant or highly correlated.
- Including Quadratic Terms LASSO selects forward citations, backward citations, and family size square terms as well.
- All covariates standardized.

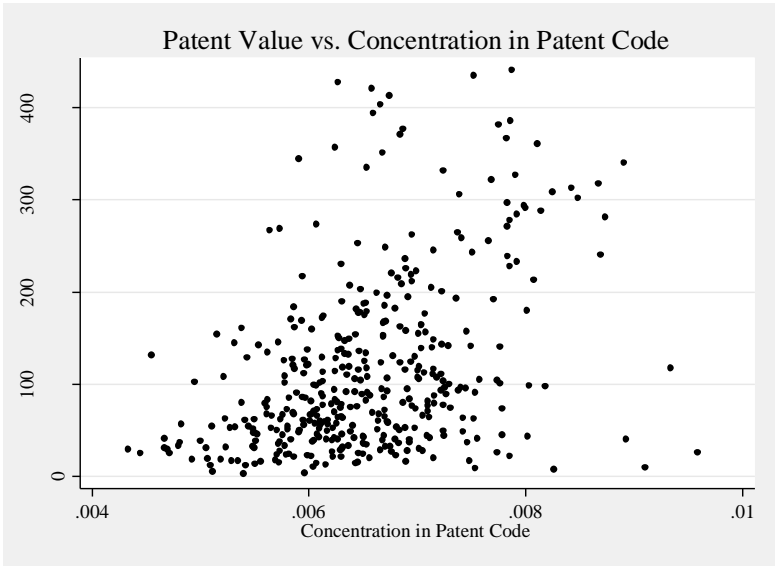
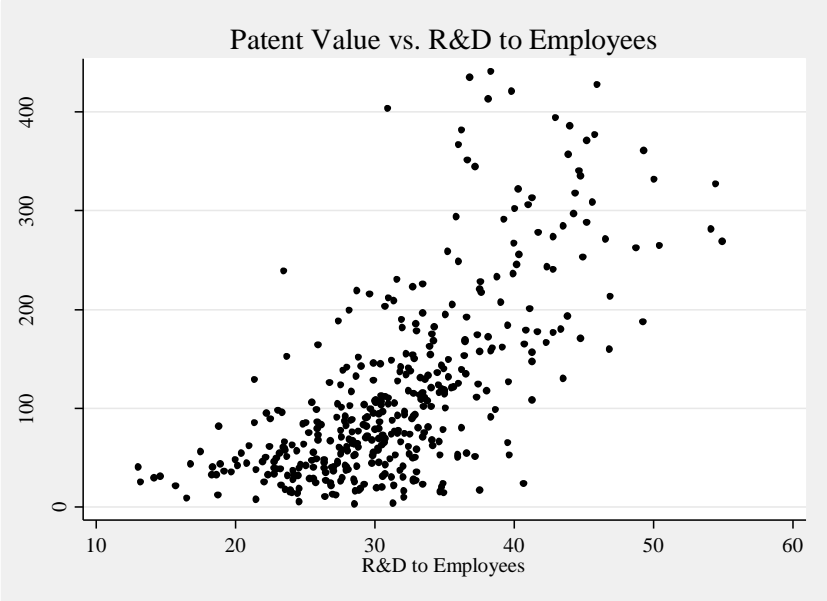
How We Construct Industry Characteristics

- **Industry data is readily available for public firms, representing 46% of our US patents**
- **Project the most likely industry for remaining patents using the patent's tech class**
 - **Innovators are still competing in the same industry**
 - **Private firms unlikely to be the dominant player in an industry**

Summary Industry Characteristics

	Mean	Standard Deviation
Industry Cocentration (HHI)	862	375
Industry Leverage	17.20%	7.98%
Industry Maturity	-1.0	5.3
Industry Market-to-Book	2.4	1.1
Industry Lifecylce Stage	3.3	1.4
Industry Profitability	-0.4%	11.6%
Industry Sales Growth	50.1%	114.1%
Industry Cash-to-Employees	103.2	81.6
Industry R&D-to-Employees	35.8	28.1
USPTO Patent Granted	888.9	2.4
USPTO Inventor Concentration	2.9%	3.3%
USPTO Technology Concentration	0.01%	0.01%

Industry Characteristics



Variable Selection

	LASSO Linear	LASSO Quadratic
Industry Concentration (HHI)	-0.060 (0.010)***	
Industry Leverage	-0.055 (0.011)***	-0.031 (0.048)
Industry Profitability	0.027 (0.010)***	0.044 (0.018)**
Industry Sales Growth	-0.014 (0.008)*	0.000 (0.008)
Industry R&D-to-Employees	0.061 (0.012)***	0.299 (0.036)***
Industry Lifecycle Stage	0.018 (0.010)*	-0.195 (0.052)***
USPTO Patent Granted	-0.041 (0.021)*	-0.036 (0.021)*
USPTO Inventor Concentration	-0.024 (0.015)*	-0.025 (0.014)*
USPTO Technology Concentration	0.039 (0.018)**	0.042 (0.018)**
Industry Maturity		0.008 (0.008)
Industry Market-to-Book		-0.005 (0.044)
Industry Cash-to-Employees		0.061 (0.034)*
Patent Characteristics	Yes	Yes
Year Fixed Effects	Yes	Yes
Technology Fixed Effects	Yes	Yes
USPTO Fixed Effects	Yes	Yes
Examiner Fixed Effects	Yes	Yes
R^2	39%	39%

* p<0.1; ** p<0.05; *** p<0.01

- Variable Selection eliminates some of the patent characteristics as redundant: Backward Citations, Recent Technology, Claims
- Citations and Citations Squared still important
- Key nonlinearity in Industry Cash-to-Employees (-), Industry R&D-to-Employees (-), and Industry Lifecycle Stage (+)

Machine Learning in Practice

- **Several Algorithms for Prediction**
 1. **Clusters**
 2. **Trees**
 3. **Classification Rules**
 4. **Functions**
 - **Functions produce stable but biased bounds**
 - **Clusters are less biased but more unstable**
 - **Models in the middle trade-off stability, bias, and size limitations**
- **Tenfold cross-validation**
 - **Akin to bootstrapping**
 - **Split the data into 10 equal partitions, each in turn is used for testing, and the rest for training, so in the end every observation used once for testing, randomize the split and repeat multiple times average error estimate.**

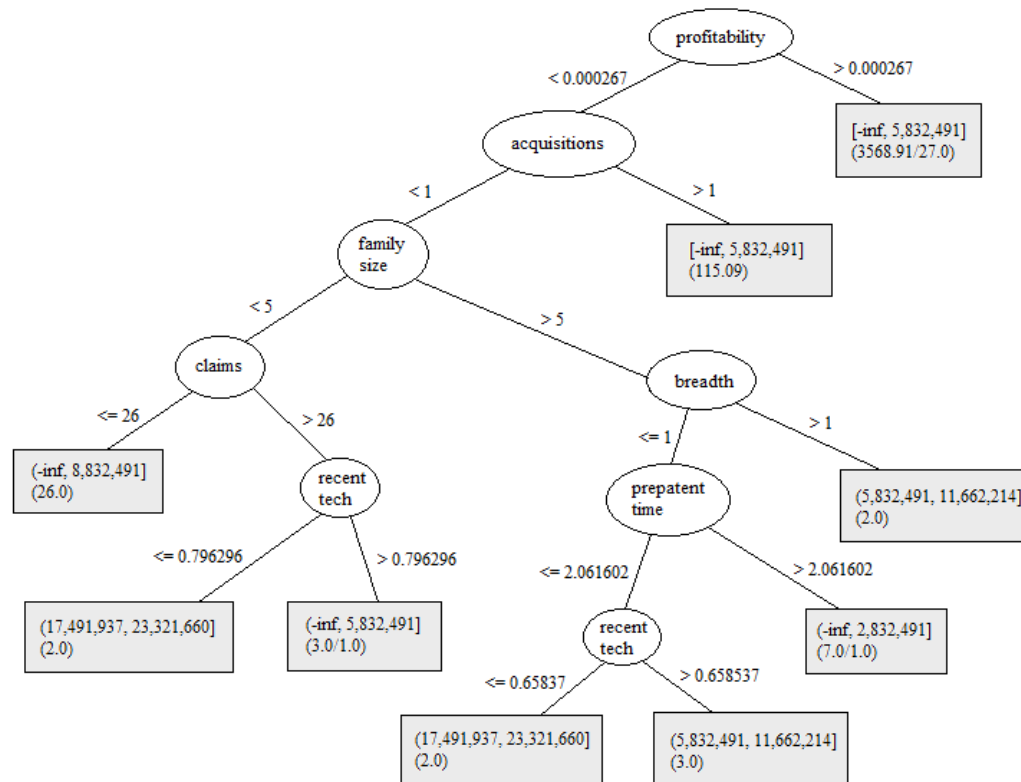
How to Compare Across Models?

- For numeric values performance measures include average of:
 - Root mean-squared error (lower is better)
 - Root relative squared error (lower is better)
 - Correlation coefficient (higher is better)

Ex. Of Decision Tree – m5p

Main steps in estimating:

- Each leaf stores the average value of obs. that reach that leaf.
- Splits are determined by minimizing the variation in the values down each branch.
- At final node, performs linear regression using all attributes, then greedily drops term if doing so improves the error estimate.



Decision Tree Results

Predicting Patent Value (\$000s)	Linear Regression	Decision Tree
Correlation Coefficient	0.361	0.475
Mean Absolute Error	65.8	59
Root Mean-squared Error	89.1	83
Root Relative squared Error	95.8%	89.2%

Decision Table selected as key variables: application year, tech category, originality, family size, backward citations, assignee type, & USPTO code.

Conclusions

Next Steps

- **New proxy for value of innovation**
- **Optimal patent policy given productive and defensive patents**
- **Value of innovation over time**
- **Value of innovation by funding type**
- **Value of innovation by entity size**
- **Value of innovation by market structure**
- **More!**

Preview of Next Talk

What Is Statistical Learning?

- **Statistical Learning is the extraction of implicit, previously unknown, and potentially useful information from data.**
- **Finding strong patterns help to make accurate predictions on future data.**
- **The goal is to find an algorithm robust enough to cope with imperfect data and imprecise patterns.**

Statistical Learning in Practice

Step 1. Create Decision Bounds

– 4 ways to represent patterns and create bounds

1. Clusters

2. Trees

3. Classification Rules

4. Functions

– Functions produce stable but bias bounds

– Clusters are less bias but more unstable

– Models in the middle trade-off stability, bias, and size limitations

Statistical Learning in Practice

Step 2. Determine Prediction Error as Function of Model Complexity

- Divide data into a training and testing set
- Estimate model parameters from training set
- Estimate prediction error from test set

Tenfold Cross-Validation – Split the data into 10 equal partitions, each in turn is used for testing, and the rest for training, so in the end every observation used once for testing, randomize the split and repeat multiple times average error estimate.

Statistical Learning in Practice

Step 3. Compare the Performance of Different Statistical Learning Scheme

- For numeric values such as patent value, typical performance measures include:
 - Root mean-squared error (lower is better)
 - Mean absolute error (lower is better)
 - Root relative squared error (lower is better)
 - Relative absolute error (lower is better)
 - Correlation coefficient (higher is better)

Extra Slides

Additional Portfolio Characteristics

	Mean	Std. Dev.	25th	Median	75th
Claims	20.1	16.1	10	17	25
Dependent Claims	16.4	14.4	7	14	20
Inventors	2.1	1.5	1	2	3
Family Size	12.1	61.1	1	3	5

Model: Assumptions for **Productive** Innovations

- Innovations come in technology clusters
- A technology class starts with a radical innovation that has a value η
- Subsequent follow-on innovations build on this radical innovation in the same technology cluster.
- Innovations run into diminishing returns within the cluster: n^{th} innovation has a value $\eta\alpha^n$ where $0 < \alpha < 1$.
- Each new innovation cites the previous patents within the same cluster to acknowledge that they are technologically related.

Model: Assumptions for **Defensive** Innovations

- Incumbents can pay fixed cost $\psi > 0$ and produce defensive patent to protect an earlier productive patent
 - Fixed cost implies that you want to protect only the high value productive patents.
- A defensive patent increases the cost of innovation for the subsequent innovators by a *random* factor $m > 1$.
- Intuition: accounts for uncertainty in validity and efficacy of defensive patent
 - Hence a defensive patent that generates higher m has a higher defensive value.
 - At the same time, attracts less entry and receives fewer citations.

Revenue Allocation

- Patent-year-customer level data
- Patents assigned rank (1 - 4) based on negotiations with customers
 - Rank 1 most heavily relied up on in negotiations
 - Rank 4 least relied upon
 - Objective (but confidential) criteria used to determine Rank
- Rank 1 assigned higher percentage of revenue collected, Rank 2 assigned less, etc...
- Aggregate across all customers to get patent-year observations

Revenue Regression Results – Full Sample

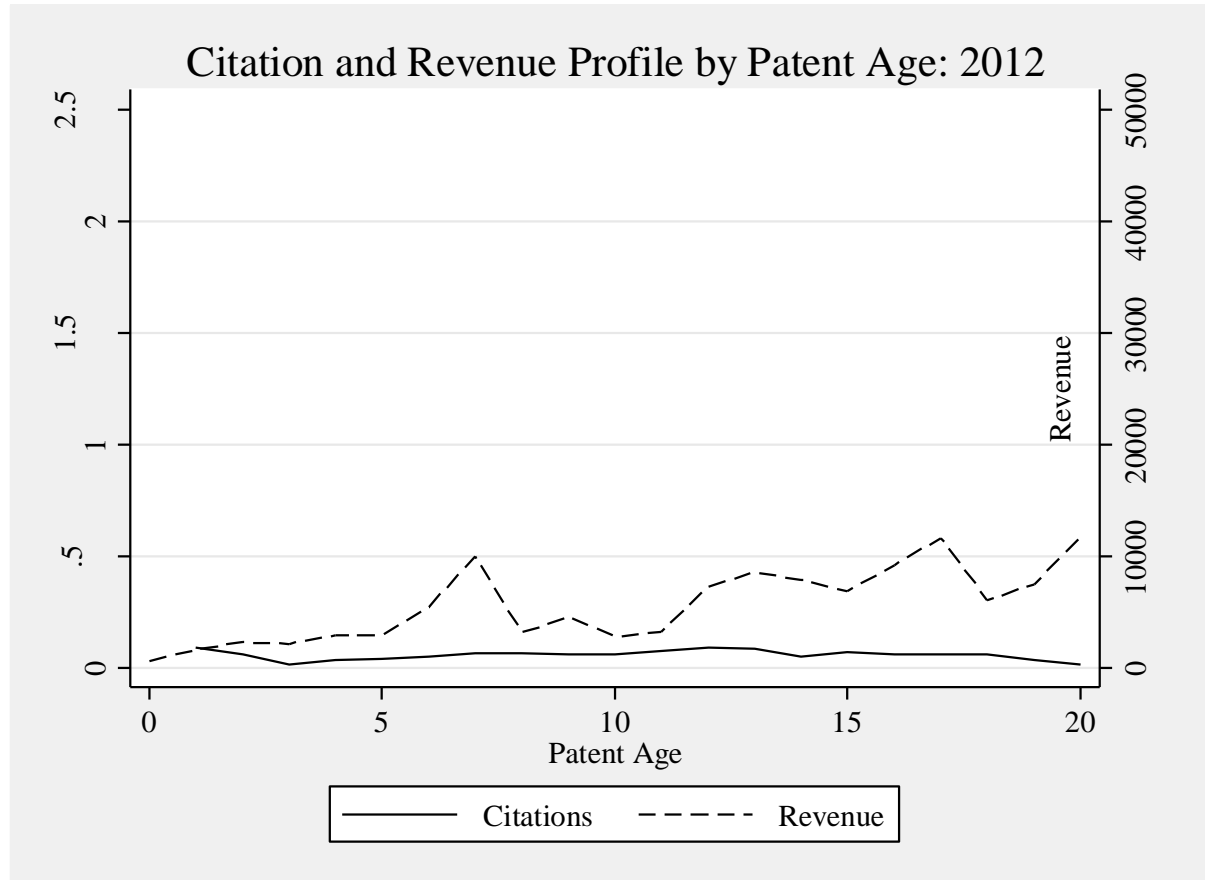
Covariate	Coefficient
Patent Age	3,058 *** (1,030)
Claims	861 *** (375)
Dependent Claims	-416 (457)
Inventor	-1,410 *** (676)
Family Size	-19 (14)
Breadth	-3,965 *** (1,128)
Indepth	395 (453)
Reissue	5,431 (5,797)
U.S.	7,086 *** (2,272)
Original	-1,236 (3,208)

Covariate	Coefficient
Patent Pendency	1,789 *** (469)
Forward Citations	357 *** (88)
Backward Citations	12 (9)
Recent Technology	5,675 (3,933)

Categorical Covariates	Joint F-test
U.S. Class Code	3.2***
Technology	8.1**
Treaty	0.5
Acquisition Method	5.8***
Year	70.4***

*Normalized such that the mean revenue per patent per year is \$10,000.

Incremental Revenue and Citations in 2012



***Normalized such that the mean annual revenue per patent is \$10,000.**

$$\text{Patent Value} = \alpha \text{Citations}^\beta$$

Technology	Estimated Alpha	Estimated Beta	Value with 3 Citations	Value with 15 Citations
Internet & Software	33,300	0.32	47,345	79,280
Computer Architecture	60,227	0.18	73,486	98,356
Optical Networking	28,517	0.14	33,088	41,140
Semiconductor Devices	47,120	0.12	53,710	65,061
Wireless Communications & Computing	50,258	0.11	56,962	68,430
Networking & Communications	63,206	0.09	70,005	81,309
Electro-Mechanical	41,953	0.04	43,848	46,779
Peripheral Devices	47,407	0.03	49,073	51,621
Circuits	112,128	0.03	115,865	121,566
MEMS & Nano	55,962	-0.03	54,019	51,294

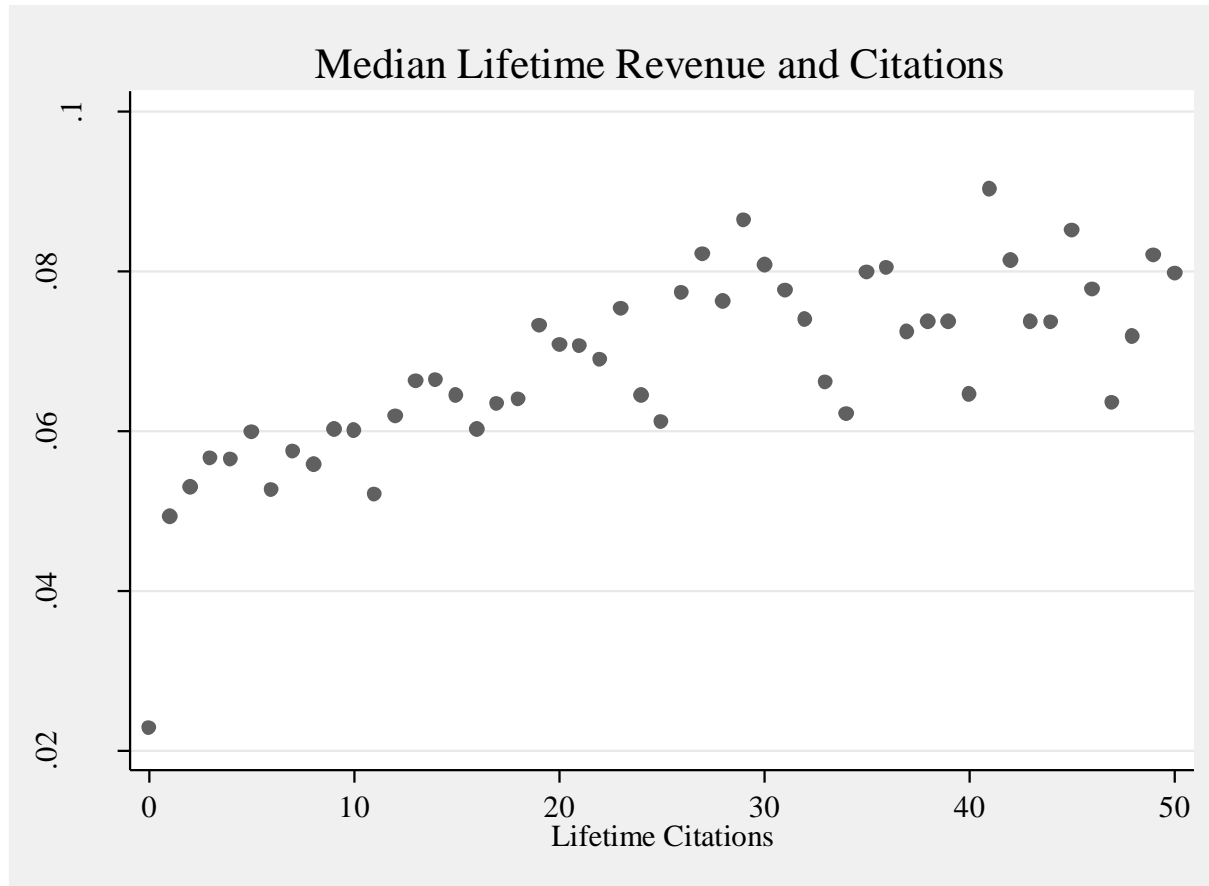
*Normalized such that the mean annual revenue per patent is \$10,000.

$$\text{Patent Value} = \alpha \text{Citations}^\beta$$

Technology	Estimated Alpha	Estimated Beta	Value with 3 Citations	Value with 15 Citations
Internet & Software	29,895	0.35	43,681	76,129
Computer Architecture	55,981	0.20	69,753	96,270
Optical Networking	26,595	0.16	31,554	40,537
Semiconductor Devices	44,807	0.13	51,830	64,154
Wireless Communications & Computing	48,035	0.13	55,145	67,503
Networking & Communications	61,153	0.10	68,327	80,385
Electro-Mechanical	41,402	0.04	43,422	46,562
Peripheral Devices	46,548	0.04	48,476	51,445
Circuits	110,973	0.03	114,979	121,111
MEMS & Nano	56,969	-0.04	54,679	51,489

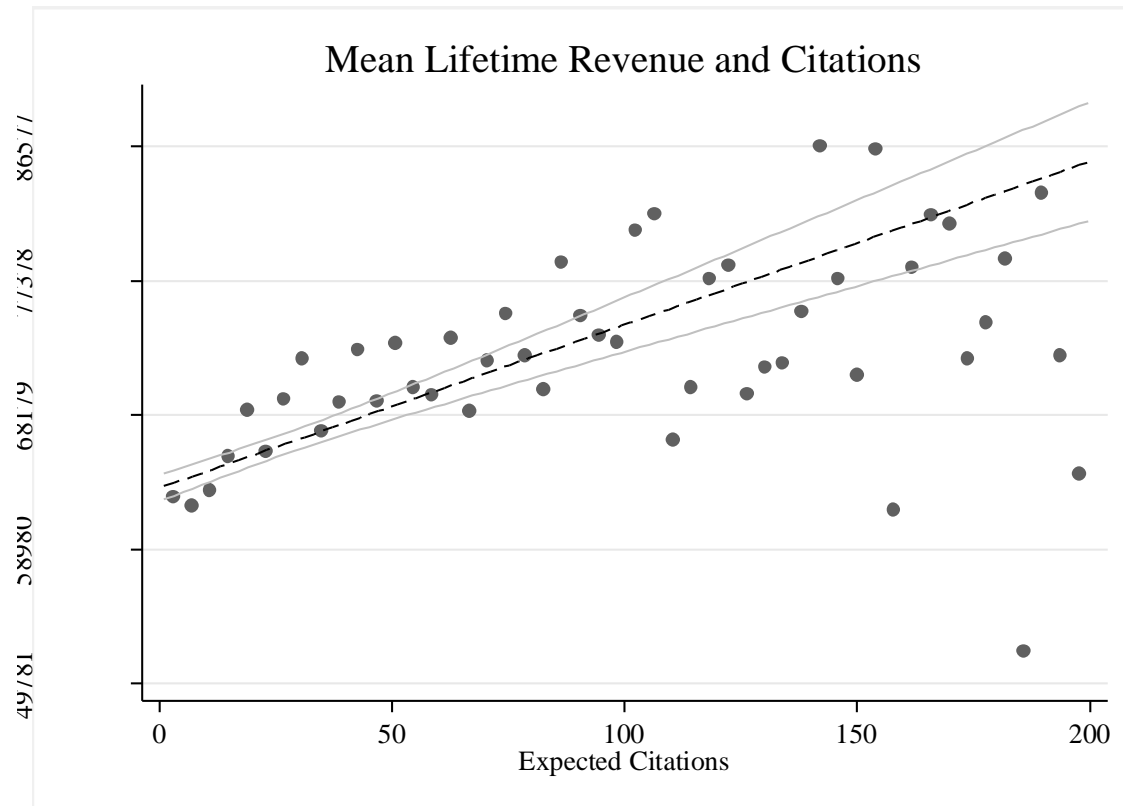
*Normalized such that the mean annual revenue per patent is \$10,000.

Median Lifetime Revenue and Citations



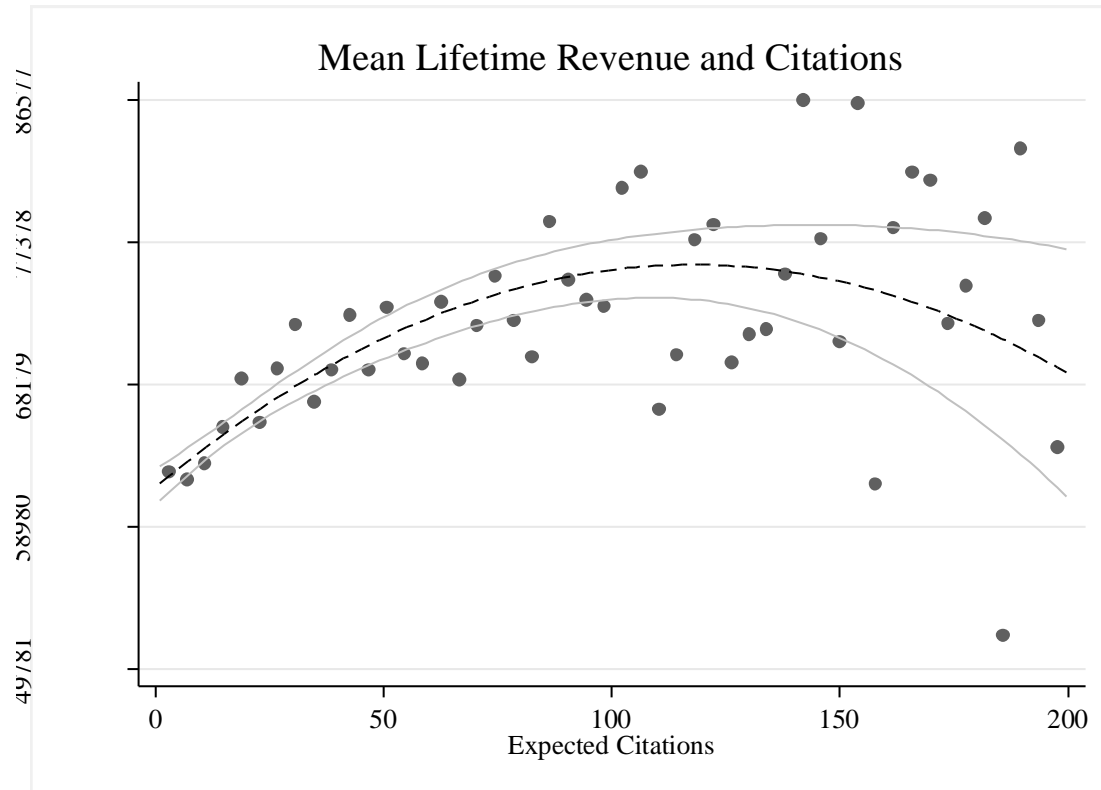
***Normalized such that the mean annual revenue per patent is \$10,000.**

Linear Approximation



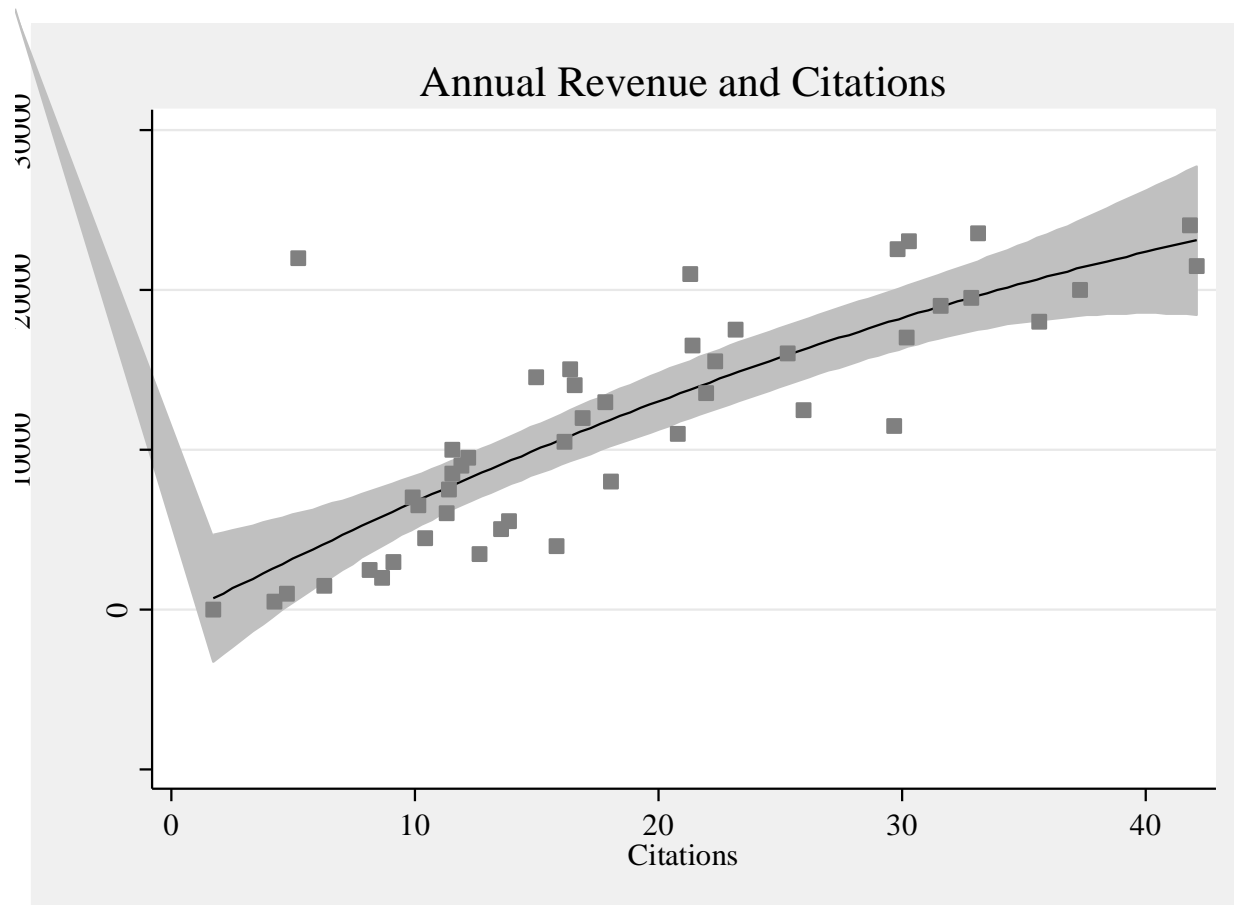
***Normalized such that the mean annual revenue per patent is \$10,000.**

Quadratic Approximation

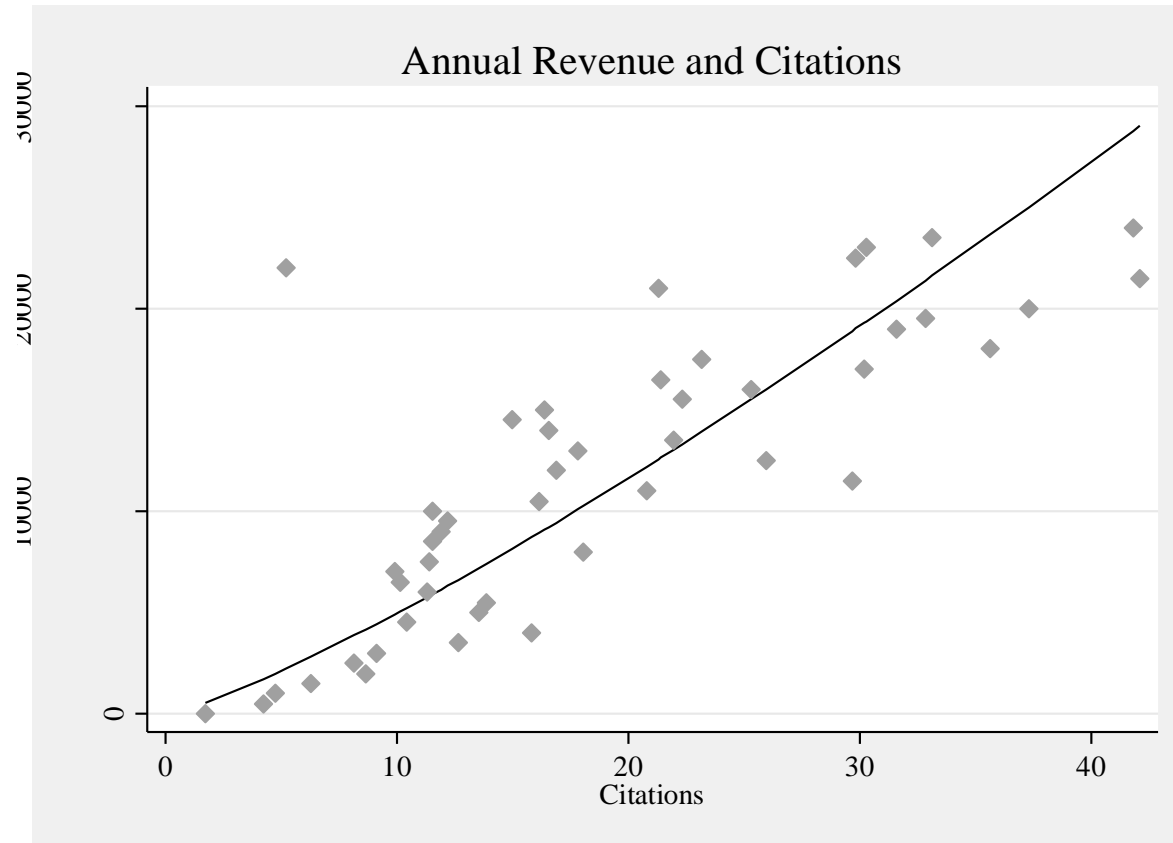


***Normalized such that the mean annual revenue per patent is \$10,000.**

Quadratic Approximation Annual Revenue



Power Law Approximation Annual Revenue



Future Work

- **New Model of Patent Value**
 - Move beyond citation-weighting to develop a model for predicting individual patent value early in patent life-cycle.
 - Approach is to use statistical learning methods such as LASSO, Ridge, Spike and Slab, and Bayesian models.
 - Examine fit for all patents and extreme tail of distribution.
- **Patent value by Technology**
 - Explain variation in patent value across industry by incorporating market size, elasticity and quality of patented vs. non-patented innovations, and other strategic components.

Broad Range of Inventors

**INDIVIDUAL
INVENTORS**

**SMALL
FIRMS**

**LARGE
FIRMS**

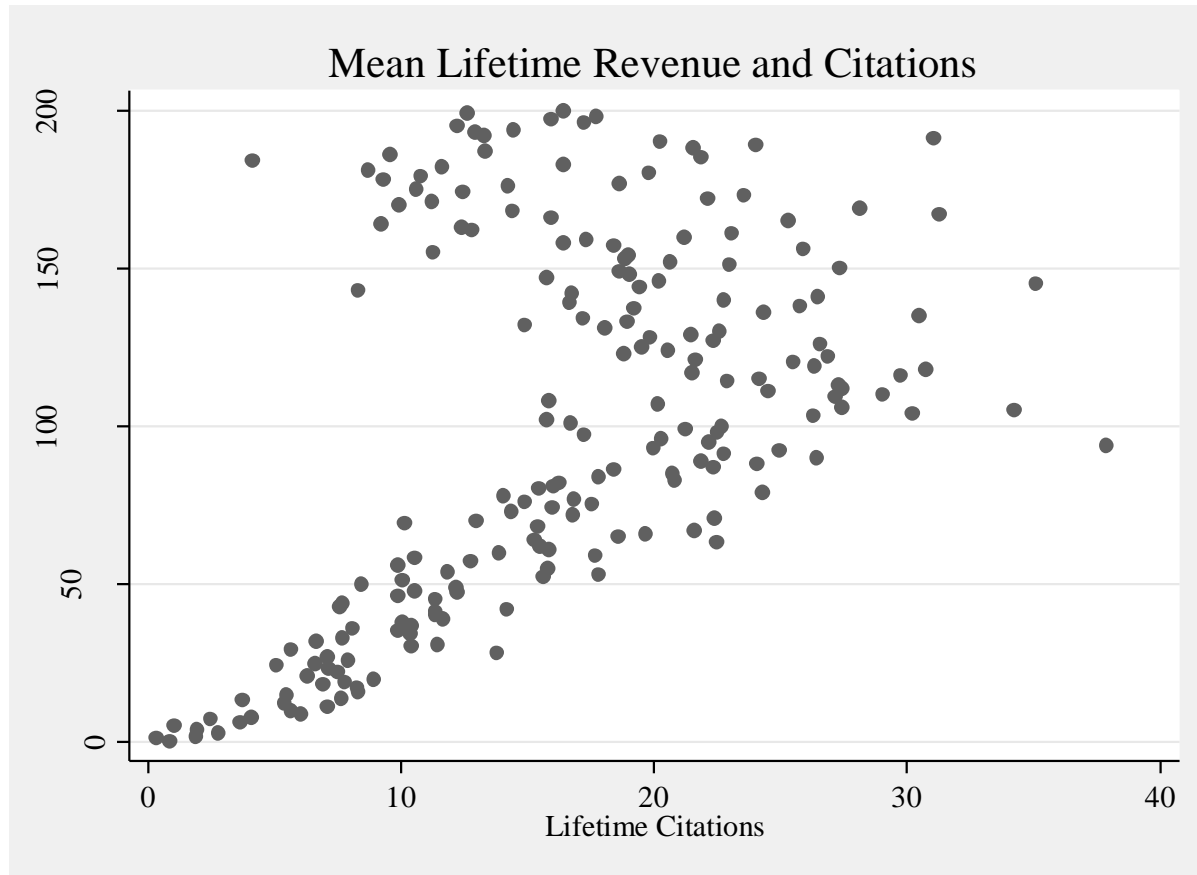
**UNIVERSITIES
AND
HOSPITALS**

GOVERNMENT

Most Patents are in Tech

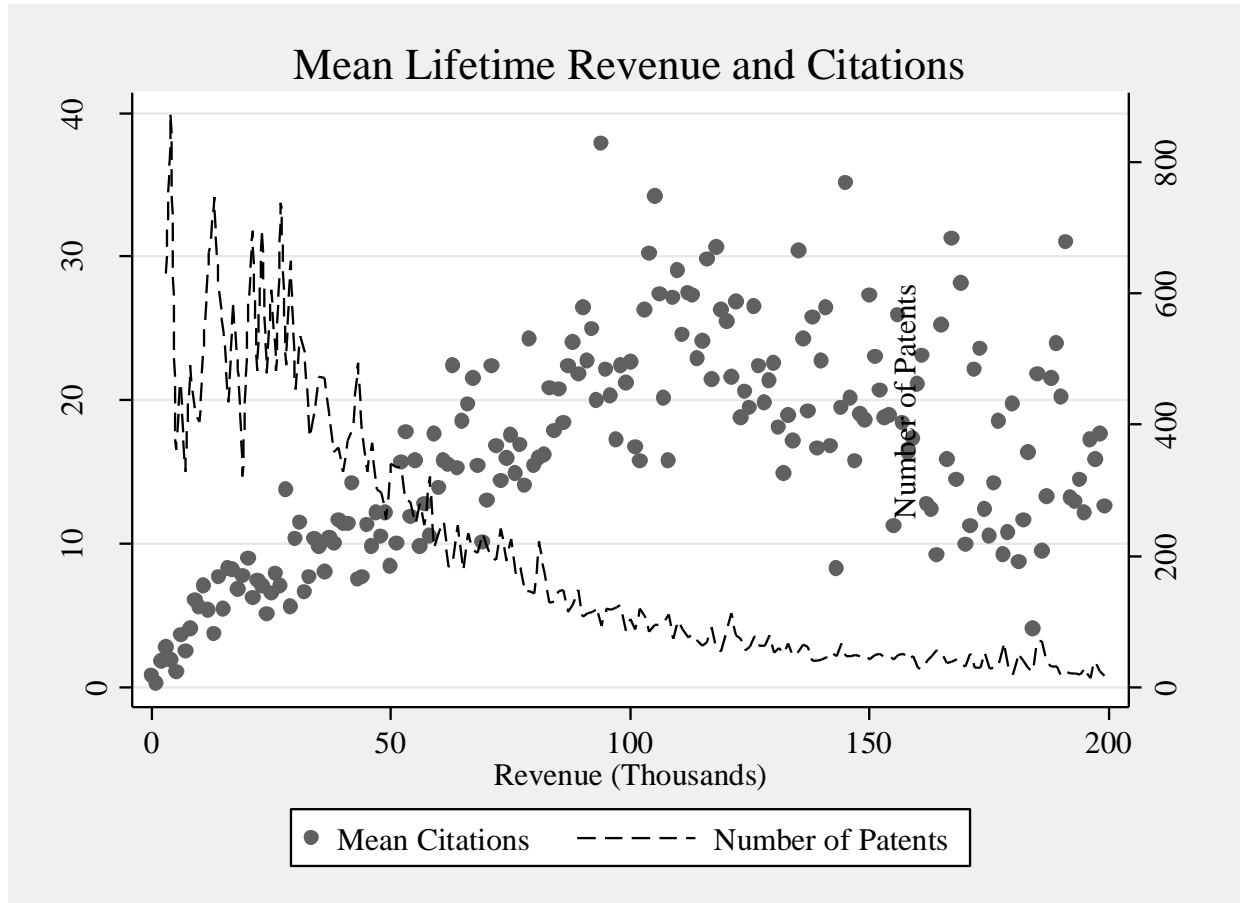
- **Primary technologies are (roughly equal size):**
 - Internet and software
 - Peripheral devices
 - Semiconductors
 - Wireless communication
- **Followed by:**
 - Circuits
 - Computer architecture
 - Networking communications
 - Optical
- **With fewer patents in:**
 - Electro-mechanical
 - MEMS & Nano-technologies

Lifetime Revenue and Mean Citations



***Normalized such that the mean annual revenue per patent is \$10,000.**

Lifetime Revenue, Citations and Observations



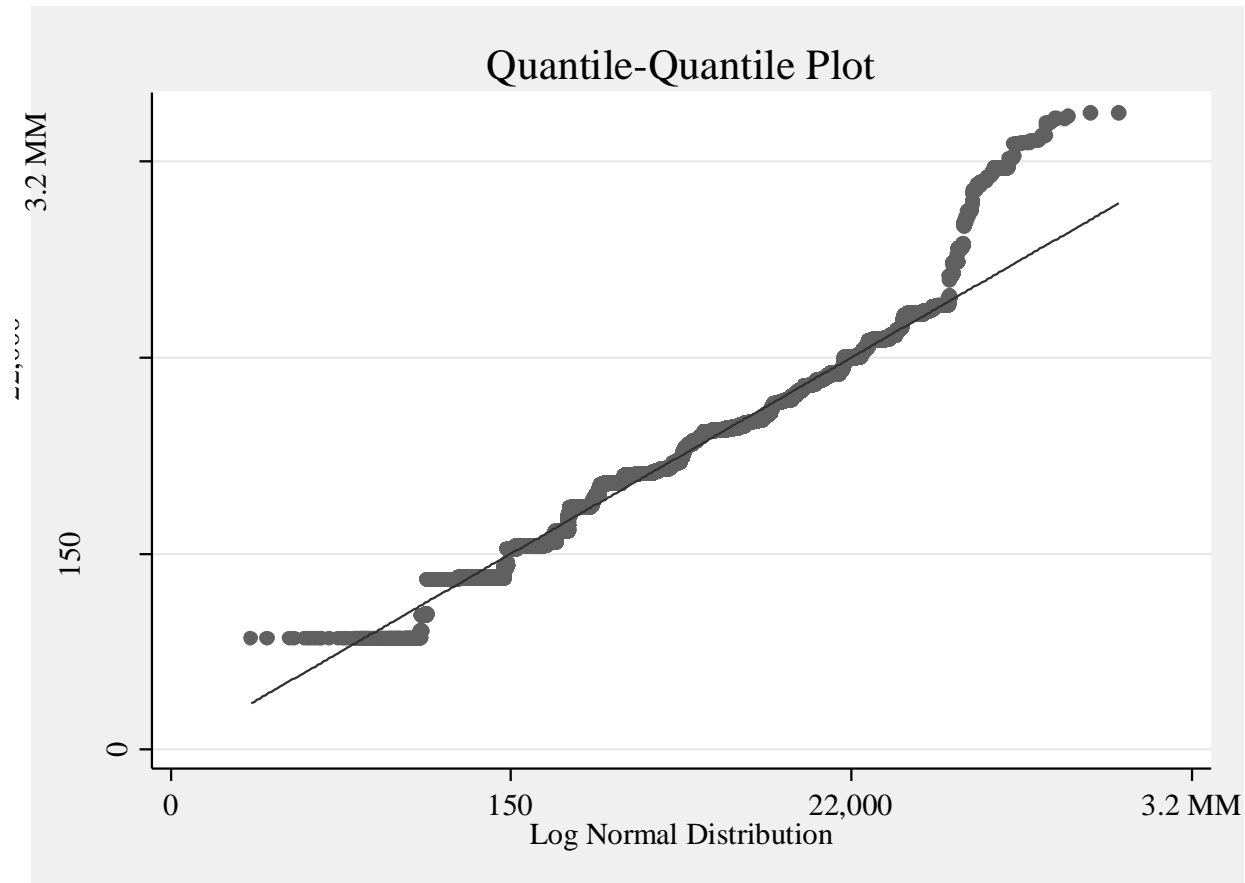
***Normalized such that the mean annual revenue per patent is \$10,000.**

Power Law Regression Results

$$\text{Patent Value} = \alpha \text{Citations}^{\beta}$$

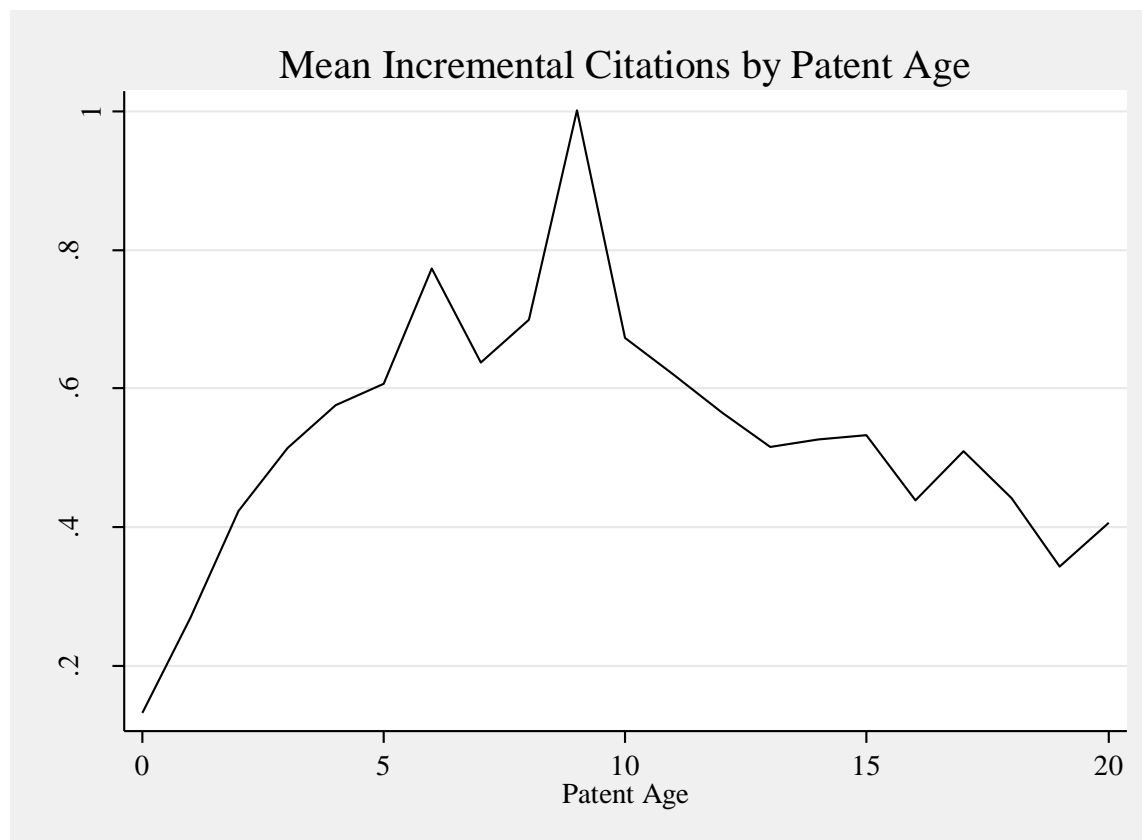
	Standard Error	T-stat	Lower 95%	Upper 95%	F-stat	
1.34	0.09	14.64	1.16	1.52	214.43	63%

Patent Revenue Has a Fat Tail

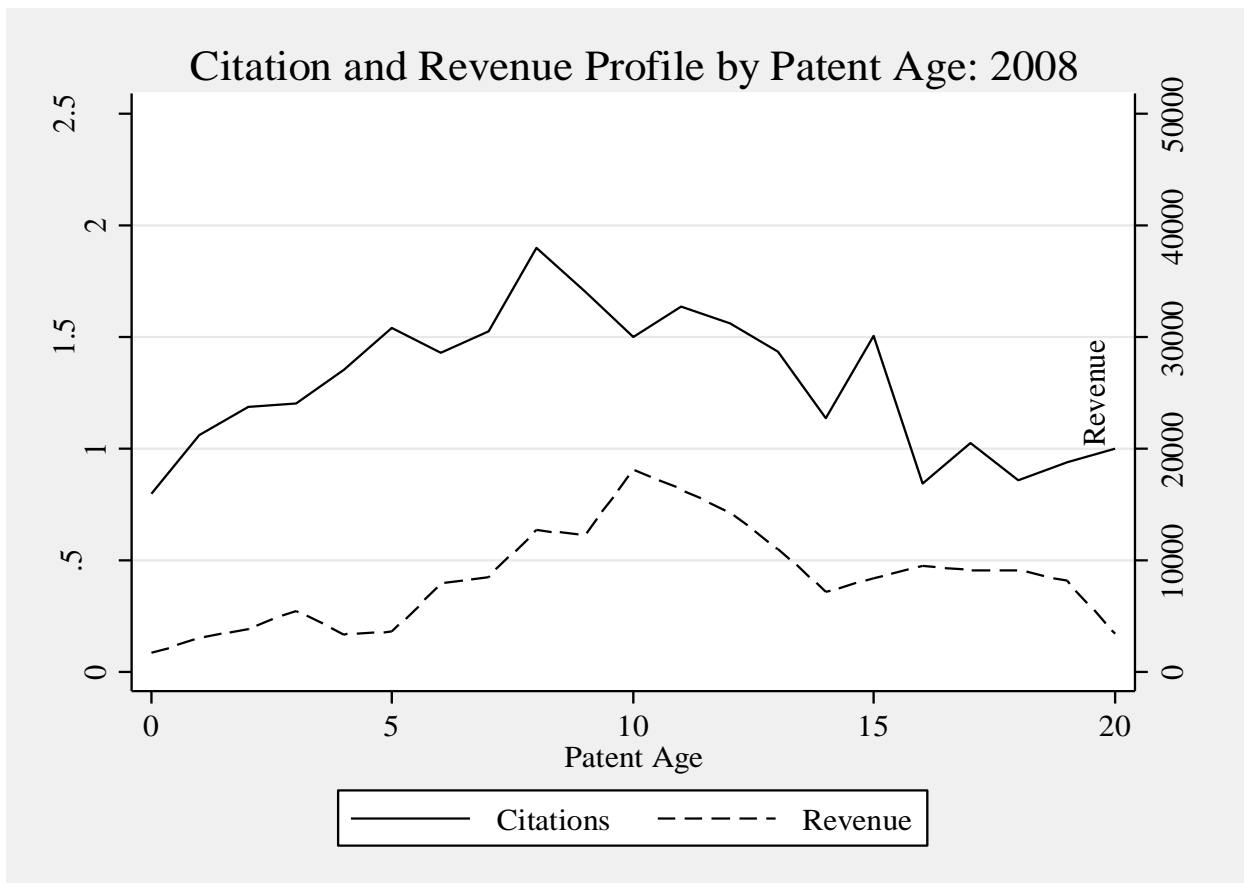


***Normalized such that the mean annual revenue per patent is \$10,000.**

Incremental Forward Citations vs Age

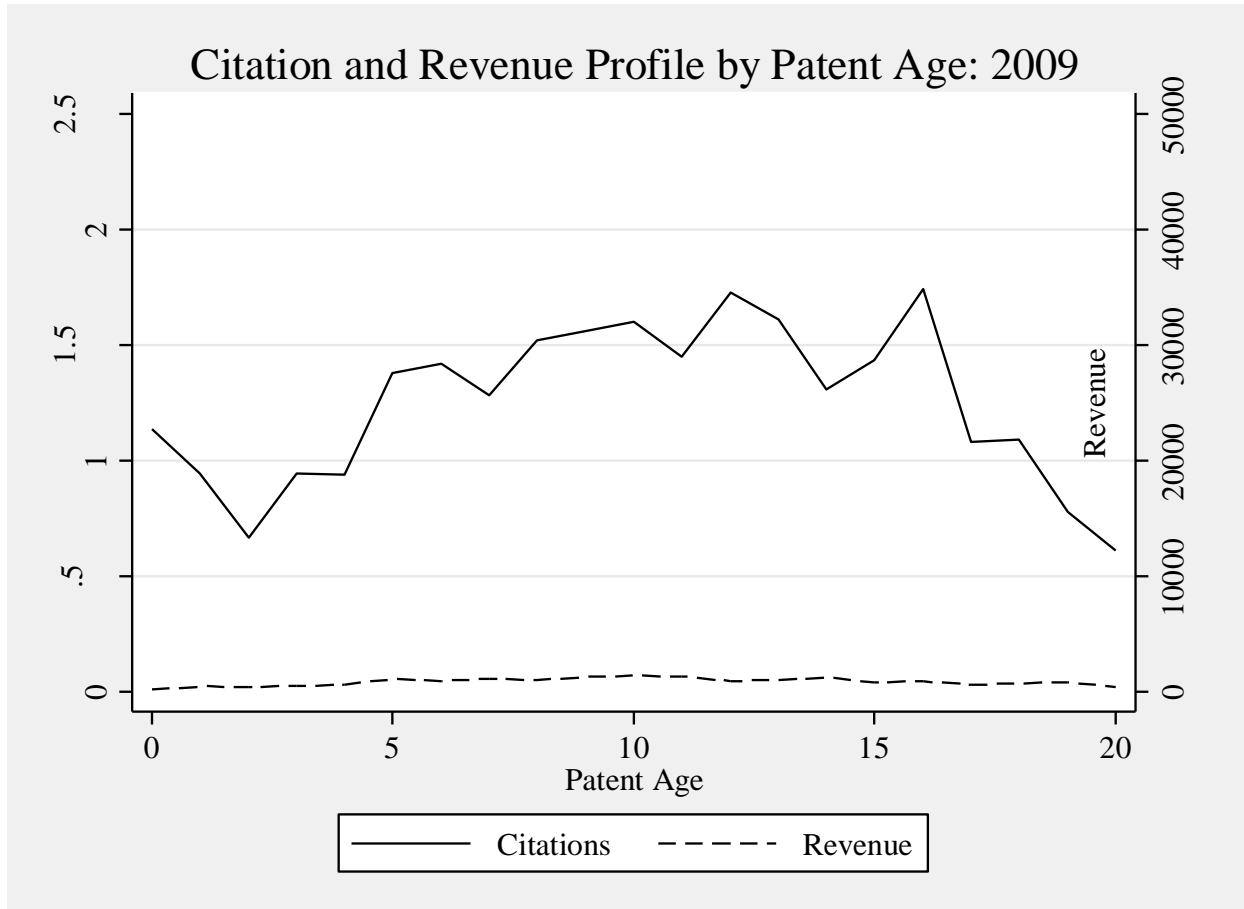


Incremental Revenue and Citations in 2008



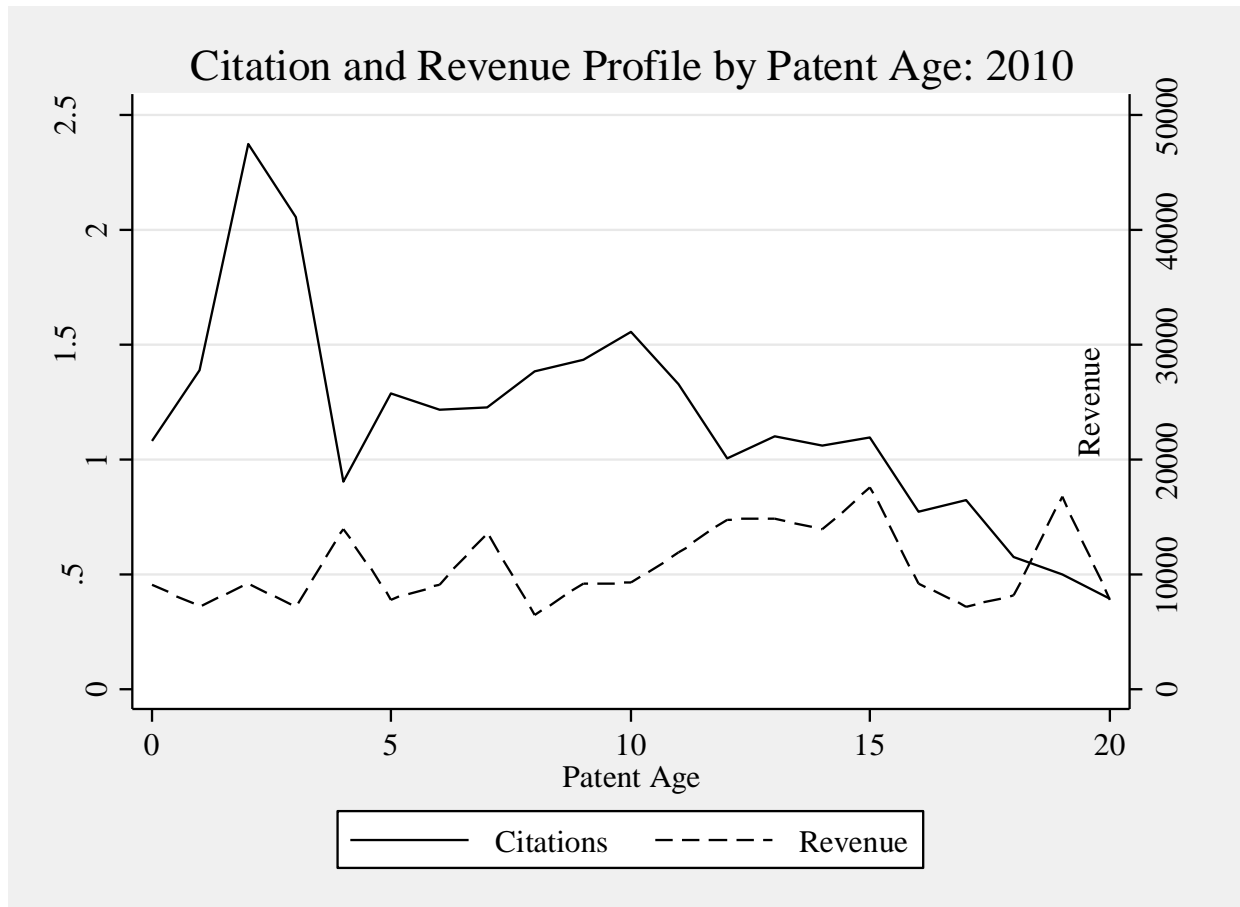
***Normalized such that the mean annual revenue per patent is \$10,000.**

Incremental Revenue and Citations in 2009



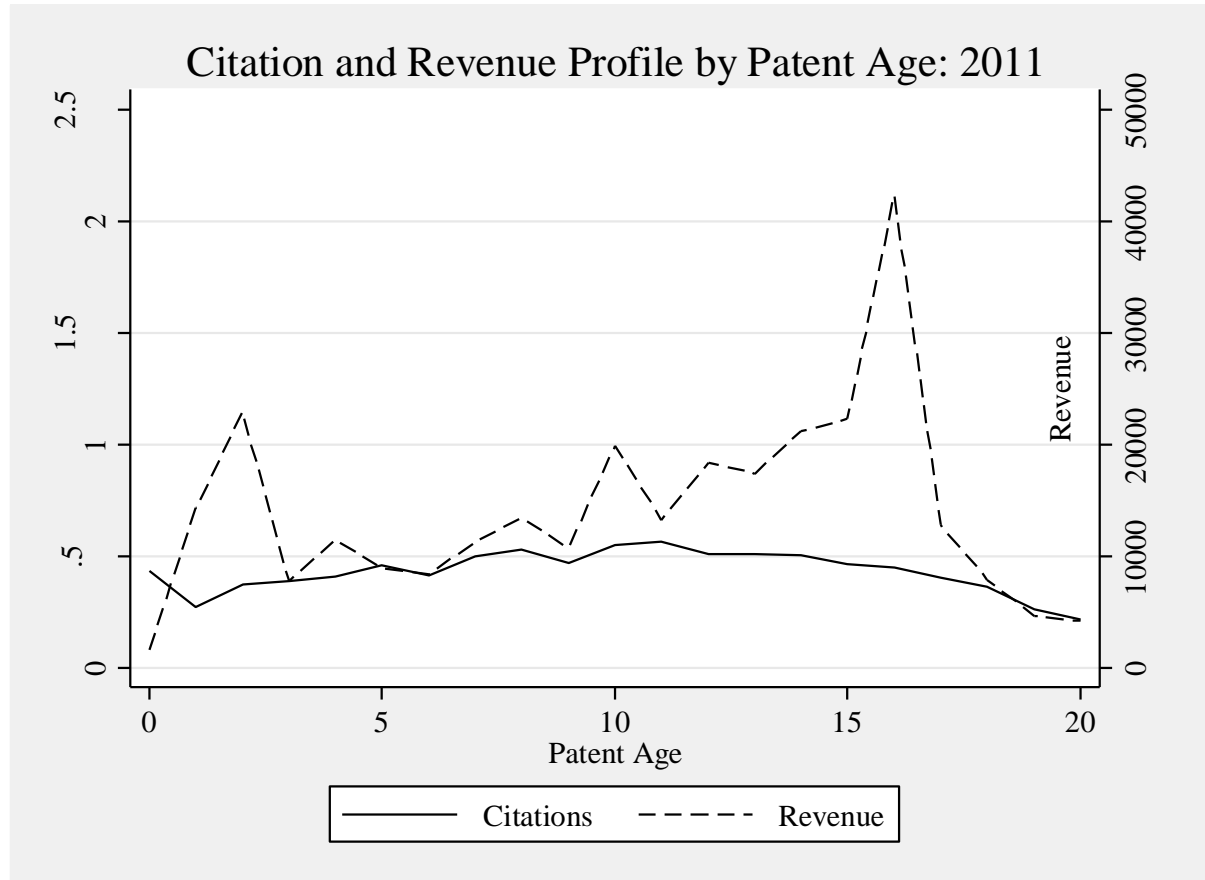
***Normalized such that the mean annual revenue per patent is \$10,000.**

Incremental Revenue and Citations in 2010



*Normalized such that the mean annual revenue per patent is \$10,000.

Incremental Revenue and Citations in 2011



***Normalized such that the mean annual revenue per patent is \$10,000.**