VALUING PATENTS USING RENEWAL DATA : AN INQUIRY INTO THE FEASABILITY OF AN AUTOMATED PATENT SCORING METHOD[1]

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[1] This paper benefited from valuable discussions with Mark Schankerman. Remaining errors are ours.

ABSTRACT

In this paper, we address the problem of patent valuation. With this aim in view, we focus on the feasibility of a patent rating system. This leads us to develop a structural model of patent renewal decisions based on real options that links patent renewals and patent value and to estimate it on micro level data. Results for a sample of European patents show that, in spite of the existence of significant impacts of some patent metrics, unobserved heterogeneity is too high to efficiently discriminate patents according to their private value. Our results thus cast some doubt on the possibility to develop a reliable rating system based only on patent metrics.

1. INTRODUCTION

Intellectual property rights (IPRs) have today reached a level of financial and strategic visibility that offers the possibility of an emerging class of IP based-transactions. Managed for a long time like a simple tool for protection, they are now used more and more like a primary source of value creation. In the current competing context, companies are indeed encouraged to manage their IPRs, not only in a defensive way but also as a tool of financial valorization. Whereas, traditionally, companies saw in IP laws a budget item and a simple legal mean to protect their investments from competitors, much of them now consider IPRs and, in particular patents, as a source of competitive advantage and as an important constituent of their capacity to create value and attract external financing[2].

The valuation of assets concerned with IPRs corresponds without any doubt to a true need, for companies as for the financial sphere. Indeed, from an investor's point of view, a correct understanding of the drivers of patent valuation may favor a better allocation of capital and a reduction of investment risks. However, the process by which the management of IPRs becomes a major source of value for companies is still in its infancy and the evaluation of IPRs continue to butt against serious difficulties linked to the lack of generally accepted methodologies of valuation and the fact that it is difficult to allot a value to a patent at the time of its deposit or shortly after.

Patents do not let themselves be evaluated in a univocal and indisputable way. There are indeed many legal, technical and market factors that have to go into assessing a patent's

[2] By acting as "quality signals", patents can be used as collaterals for technology-based firms to attract external financing. Even if they are still in an early stage of development, patent backed financial instruments like patent loans, patent sale and lease-back or patent securitizations can be used by companies to leverage their most valuable assets to finance their development.

overall value. However, some new statistical benchmarking methods are emerging which are supposed to ease the task of working out patent quality. This is for instance the case of the Intellectual Property Quotient (IPQ®) built upon patented methodologies by *Ocean Tomo Patent Ratings®* or the Pantros IP patent Factor Index analytics (PFI)™ developed by *PatentCafe®*. These proprietary ranking or scoring methods use multivariate regression modelling to essentially predict the probability of maintenance or abandonment of patents based on a number of identified predictor variables, also called patent metrics. However, these proprietary methodologies are not exempt from criticisms. First, the main challenge when it comes to ensure a correct valuation of patents is to develop a methodology that both buyers and sellers can trust. In this respect, arguments by providers of patent scoring and rating systems saying that their algorithms are "proprietary", and therefore cannot be disclosed, disregard the critical requirement of transparency. Second, from a more theoretical point of view, the methods proposed by these rating agencies postulate an unambiguous link between patent duration and patent value[3], i.e. the value of the patent right. In opposition to this view, we postulate that both aspects are certainly related but that the relationship is not as simple as suggested by these rating agencies. Formally, inferring patent value from maintenance or abandonment decisions requires the development of a structural renewal decision model. *Adhoc* specifications of patent duration that seem to be used by patent rating agencies are only reduced forms of such a structural model. While they enable to draw a link between patent duration and patent value, they do convey a significant risk of error.

Generally speaking, it is obvious that in order for any statistical patent evaluation process to be trustworthy, the process must be completely transparent, repeatable and objective. With this aim in view, our paper describes and implements an original approach for comparatively rating and benchmarking patent performance using statistical modeling based on an analysis

[3] For a discussion on what it is meant by the term « patent value », see Hall (2009).

of observed renewal decisions conditional on some patent metrics. Several arguments play in favor of an analysis based on patent renewals. First, significant insights can be gained from analyzing past renewal decisions of patent owners. Indeed, for a patent to be kept in force, renewal fees have to be paid on an annual basis in Europe versus every 4 year in the U.S. by all patent owners whatever their status. In most cases, it is fair to assume that patent owners are uniquely knowledgeable and well-qualified to make internal patent value and risk assessments. They are also economically motivated to make timely and relevant assessments and to take sound decisions based thereon. This means that they will choose to pay renewal fees only when the perceived value of the expected remaining economic benefit secured by the patent exceeds the amount of the maintenance fee. Thus renewal decisions indirectly reveal the implicit value of a patent. By contrast, methods examining the relationship between patent counts/sales and Tobin's q (Hall, 1993; Hall & alii, 2005; Bessen, 2009) or event studies approaches (Austin, 1993) are more restrictive in the sense that they can only be implemented on patents applied for by firms listed on the stock market and are more debatable in the case of event studies. Objectivity of the assessment method is also an important requirement. In this respect, methods based on revealed value have to be preferred to those based on stated value. Patent renewal methods belong to the first category whereas, for instance, survey methods (Harhoff & alii, 2003) belong to the second category. Moreover, as determinants of patent renewals are restricted to observed patent metrics and variables related to the macro-level context, the third requirement of repeatability is also fulfilled.

Originally developed by Schankerman and Pakes (1986) on aggregated renewal data, models of patent renewals are not as such implementable for individual patent scoring as they rely on aggregated renewal data and on the hypothesis that the rent is purely determinist. By contrast, option models proposed by Pakes (1986) and Baudry & Dumont (2006) insist on the stochastic dynamics of the rent but are still estimated on aggregate data. Moreover, such models have the disadvantage to be complex to estimate and micro-level explicative variables are difficult to introduce. To our knowledge, the only article that tries to extend the method to micro-level renewal data is Bessen (2008) but again the approach encompasses two drawbacks: first, the depreciation rate of the rent is invariant and as such is not suitable to capture the effect of variables that change with patent age. This means that it is impossible to revise the estimation of patent value according to new information affecting the dynamics of the rent during the patent life. Second, the possibility of random chocks affecting the dynamics of the rent that generates the option value is not taken into consideration. Yet, as outlined by Oriani and Sereno (2011), there are multiple sources of uncertainty that generate a patent option value. It is thus a key issue to correctly address their role in patent valuation.

The originality of our paper is to propose a structural model of patent renewals decisions that links patent renewals and patent value and that can be estimated on micro-level data with observed and unobserved heterogeneity affecting both the initial rent and its dynamics. The model is flexible enough to capture the influence on patent value of both the characteristics of the patent known at the application date and those that change with patent age. By contrast with previous attempts to assess patent value at the micro level, our model also incorporates an option value of patents. Results for a sample of EPO patents designating France show that some key observable patent characteristics significantly affect patent value. Nevertheless, unobserved heterogeneity is too high to efficiently discriminate among patents. Our results show that the most significant effects are those affecting the dynamics of the rent, not the initial rent itself. This means that patent maintenance/abandonment decisions can only be observed *ex-post*. In return, this reflects the fact that uncertainty at the time of issue is very large. More fundamentally, although an automatic scoring is intellectually stimulating in itself, our results cast some doubt on the possibility to develop a reliable rating system based only on patent metrics

The paper is organized as follows: the next section presents a model of patent renewal and discusses theoretical issues that lead us to develop a structural model of patent renewal. Section 3 describes data collection and variables. Section 4 presents estimation results for a panel of European patents designating France and applied for in 1989. It also provides automated scorings to value patents. Section 5 concludes.

2. A MICRO-ECONOMIC MODEL OF PATENT RENEWALS AND PATENT VALUE

This section first offers a simple model to analyze patent renewal decisions. We then propose a structural model of patent renewals decisions that links patent renewals and patent value and that can be estimated on micro-level data with observed and unobserved heterogeneity affecting both the initial rent and its dynamics

2.1. *MODELLING PATENT RENEWAL DECISIONS*

2.1.1. *A general approach to patent renewal decisions*

A basic but key assumption to estimate patent value on the basis of patent renewal decisions is that patent owners act rationally to maximise the value of their patent conditional on information available. More precisely, at each renewal date, a patent owner has to decide whether to discontinue or not the payment if the required renewal fees exceed the value of the patent. This option to renew exists because the payment of renewal fees is discretionary.

Costs of renewing a patent are in fact multi-facets. They encompass internal costs to assess the usefulness of the patent, enforcement costs and those corresponding to the payment of renewal fees to the patent office. In practice, renewal fees depend on the age a of the patent and are revised in the course of time. Thereafter, $f_t^a \geq 0$ denotes the fee charged at age a for a patent with application date t to be renewed up to age $a+1$. Patent offices in Europe charge increasing fees each year (i.e. $f_t^a > 0 \; \forall a \in \{0, \cdots, A\}$ where A is the statutory life limit of patents) versus every 4 years in the United States. Most renewal costs like legal expenses are not directly observed nor easily measured, except renewal fees that are published by patent offices. As a result, a common practice consists in subtracting the unobserved renewal costs from the gross rent associated to the exclusivity right conferred by a patent on all industrial and commercial applications of the patented invention. The resulting net rent for a patent of age a applied for at time t is denoted by R_t^a. Gains that accrue from renewing a patent are obtained by adding the current flow of net benefits given by $R_t^a - f_t^a$ and the expected and discounted value $E_{t+a}\left[V_t^{a+1}\right]/1+r$ of the patent at age $a+1$ where r stands for the discount rate and E_{t+a} for the mathematical expectation conditional on all information available to the patent owner at date $t+a$. Renewing a patent is optimal if and only if the associated gains are positive. Thus, at any renewal date before the statutory life limit A, the value of a patent is recursively defined by the following expression:

$$V_t^a = Max\begin{cases} R_t^a - f_t^a + \dfrac{E_{t+a}\left[V_t^{a+1}\right]}{1+r} & \text{if the patent is renewed} \\ \\ 0 & \text{if the patent is withdrawn} \end{cases} \qquad \forall a < A \qquad \text{(1.a)}$$

At the statutory life limit A, the expected future value of the patent falls to zero and the value of the patent is given by

$$V_t^A = Max\begin{cases} R_t^A - f_t^A & \text{if the patent is renewed} \\ \\ 0 & \text{if the patent is withdrawn} \end{cases} \qquad \text{(1.b)}$$

The optimal age of withdrawal for a patent is the first age such that, conditional on information available at the current time, renewing the patent generates a net loss. Formally, it is the optimal stopping time associated with the dynamic programming problem (1):

$$a^* = Inf\left\{a \in \{0,\cdots,A\};\ R_t^a - f_t^a + \frac{E_{t+a}\left[R_t^{a+1}\right]}{1+r} < 0\right\} \qquad (2)$$

Whether a^* is deterministic or random depends on assumptions about the dynamics of the rent and renewal fees. For the optimal age of withdrawal to be random, either the rent or renewal fees must be affected by unexpected shocks; the observation of which constitutes new information. In that case, some authors call V_t^a the option value of patents in reference to the real option theory that analyses irreversible decisions when facing risk or uncertainty. Pakes (1986), Lanjouw (1998) or Baudry and Dumont (2006) for instance use this terminology. The stochastic nature of the dynamics of the rent makes the determination of the optimal withdrawal date rather complex in these real option approaches to patent renewal decisions. As a result, the impact of observed patent characteristics on the decision rule is not captured and the analysis is confined to the assessment of an average value of the patent rights in a cohort of patents. Our paper tries to solve this problem by extending the method to micro-level renewal data.

2.1.2. ***A simplified renewal decision rule***

A reason for the complexity of decisions rules in real option models of patent renewals is that the rent may fall far below the renewal fee but, due to expected future positive shocks, this may be reversed in the short or medium term. Hence, comparing only the current value of the rent and renewal fees is generally not a relevant decision-rule. For such a decision-rule to be efficient, an additional Assumption 1 as regards the dynamics of the rent and renewal fees is required.

Assumption 1: The gap $R_t^a - f_t^a$ between the rent and the renewal fee decreases monotonically from an initial positive value to a possibly negative value with patent age.

A more restrictive form is generally used. It states that the rent itself monotonically decreases whereas, as observed for most patent offices, renewal fees are assumed to monotonically increase[4]. Whatever the form considered, Assumption 1 implies Proposition 1 and Corollary 1:

Proposition 1: Under assumption 1, the rent will never exceed the renewal fee once it falls below it. As a result, maintaining the patent alive is optimal if and only if the rent exceeds the renewal fee.

Corollary 1: Under assumption 1, if a patent applied for at date t is withdrawn at age a, then the rent has always exceeded the renewal fee from age 0 to age $a-1$.

Figure 1 illustrates these key results in the case of renewal fees that start at $10€$ and increase at a constant rate of 10% whereas the rent for patents A, B and C respectively starts at $90€$, $100€$ and $40€$ and decreases at a constant rate that respectively amounts to 15%, 10% and 5%. The associated optimal withdrawal age is 9 years for patent A, 12 years for patent B and 10 years for patent C. If monetary flows are discounted at a 3% discount rate, patents A, B and C are respectively worth $304{,}81€$, $458{,}61€$ and $148{,}72€$.

Insert Figure 1

[4] Pakes and Schankerman (1986) also use the same assumption but not in an option model context.

Figure 1 also highlights that patent value is tightly linked to patent duration but this link is not that simple. In Figure 1, the height separating the decreasing profiles of the rent and the increasing profile of renewal fees measures the net revenue that accrues from a patent at the corresponding age on the abscissa. The total value of a patent at the application date (age $a = 1$) is obtained by discounting and summing up the heights between the two profiles for all ages on the left of the crossing point between the two profiles. The first date on the right of the crossing point corresponds to the patent withdrawal date. Due to discounting, the total value of two patents cannot systematically be compared graphically. Nevertheless, simple cases that can be easily compared are considered on Figure 1. For instance, patent B has a rent that exceeds the rent of patents A and C and this rent also lasts longer than for patents A and C. This leads us to conclude that patent B has the highest total value. Patent A is withdrawn slightly before patent C but its rent is far higher than that of patent C at almost all dates before the withdrawing dates. Therefore one can reasonably assert that the total value of patent A exceeds that of patent C. Thus, patent ranking from the earliest non-renewal date to the latest is A-C-B but the ranking from the lowest total value to the highest is C-A-B. Obviously, both rankings depart. A structural model of patent duration is thus required to derive estimates of patent value from observed characteristics affecting the date of patent withdrawal and then to correctly rank patents in terms of economic value.

2.2. *A STRUCTURAL MODEL OF PATENT DURATION*

2.2.1. *Basic specification with a constant depreciation rate*

The traditional solution adopted in seminal papers on patents valuation to account for patent heterogeneity in terms of the observed date of withdrawal, consists in assuming that patents only differ in their initial rent at the filing date. By contrast with these approaches, we assume that the rent decreases at a same decay rate for all patents so that the simplified decision rule described in Proposition 1 applies. As a result, given the initial value of the rent, the optimal withdrawal age defined in (2) is deterministic. The probability distribution of the withdrawal age is then derived from the probability distribution of the initial rent. Again, the basic specification proposed here slightly departs from previous approaches. Indeed, the probability distribution of the initial rent is conditioned on patent characteristics. This aims at introducing a source of observed initial heterogeneity between patents that explains differences in terms of expected withdrawal age at the application date. Patent characteristics influencing the probability distribution of the initial rent have to be time invariant to be consistent with the idea that they capture differences in the initial conditions influencing both the duration and value of a patent. The corresponding variables are thus referred to as static variables. Indicators of the technology field, number of claims, backward citations, patent family size are variables that typically fulfil this condition.

Formally, the value $R_{t\,i}^{a+1}$ of the net rent at age $a+1$ for a patent i applied for at date t is written as the value of the net rent $R_{t\,i}^{a}$ at the previous age, affected by a decay or depreciation factor δ_t^{a+1}. Note that the index i is introduced to capture the fact that the value of the rent may be patent specific. Conversely, the decay factor is independent of the patent characteristics but may be contingent to the application date t and the age a. Proceeding recursively, we have

$$R_{t\,i}^{a} = R_{t\,i}^{0} \prod_{s=1}^{a} \delta_t^{s} \tag{3}$$

Furthermore, heterogeneity of patents as regards the initial rent $R_{t\,i}^{0}$ follows on from observed and unobserved factors. Observed heterogeneity is captured by a vector $X_i = \{x_{1\,i}, \cdots, x_{k\,i}, \cdots, x_{K\,i}\}$ of K objectively measurable characteristics of the patent that are time-invariant. Unobserved heterogeneity, for its part, is taken into account by assuming that

$R_{t\,i}^{0}$ is drawn independently for each patent from a same probability distribution, one or more parameters of which depend on X_i. Though not necessary, it is convenient to assume that observed and unobserved heterogeneity affecting the initial rent interact multiplicatively and that observed heterogeneity is correctly captured by a Cobb-Douglas functional form of the static characteristics of each patent. Accordingly, we have :

$$R_{t\,i}^{0} = \alpha_0 \left(\prod_{k=1}^{K} x_{k\,i}{}^{\alpha k} \right) \varepsilon_i \tag{4}$$

where α_k (with $k \in \{0,\cdots,K\}$) are parameters and ε_i is a i.i.d random term. The probability distribution of the initial rent directly follows on from the probability distribution of ε_i that captures unobserved heterogeneity.

Imposing $\delta_t^a \in [0\,,1]\,\forall t\ \forall a$ in (3) guarantees that that the rent never increases. This ensures the validity of the simplified decision rule defined in Proposition 1. Then, given that the dynamics of the rent fulfils Assumption 1, Corollary 1 implies that any patent applied for at date t which is still alive at age a satisfies the following properties:

$$R_{t\,i}^{s} \geq f_{t\,i}^{\,s} \quad \forall s \in \{0,\cdots,a-1\} \tag{5}$$

Moreover, Assumption 1 implies that $R_{t\,i}^{a-1} \geq f_{t\,i}^{\,a-1}$ is a sufficient condition for all inequalities in (5) to be satisfied. Therefore, the information revealed by observing that a patent applied for at date t is still alive at age a may be synthesised by this last condition. Combining this result with (3) and (4), we finally obtain that a patent i applied for at date t is renewed up to (at least) age a if and only if

$$\varepsilon_i \geq Q_{t\,i}^{a-1} \quad \text{with} \qquad Q_{t\,i}^{a-1} = \frac{f_t^{a-1}}{\alpha_0 \left(\prod_{k=1}^{K} x_{k\,i}^{\alpha_k} \right) \prod_{s=1}^{a-1} \delta_t^s} \tag{6}$$

Once expressed in logarithms, (6) is similar to the key condition that Schankerman and Pakes (1986) or Schankerman (1998) use to obtain their econometric model. Nevertheless, for the estimation method proposed by these authors to work, the dataset of patents has to be partitioned in such a way that the threshold value $Q_{t\,i}^{a-1}$ is identical for all patents within a same subset. Moreover, the size of each subset of patents has to be sufficiently large to obtain reliable measures of the proportion of patents withdrawn at each age. This means that this econometric method works for patent cohorts (Schankerman and Pakes 1986) or patents belonging to large technological classes (Schankerman 1998) as long as none of the characteristics that distinguish patents within a subset is used as an explanatory variable. For these reasons, we suggest an alternative econometric approach that also relies on condition (6) but that is adapted to the use of micro-level patent characteristics. For this purpose, note that Assumption 1 implies that the value of the threshold $Q_{t\,i}^{a-1}$ decreases with age a. Furthermore, a patent i applied for at date t is optimally withdrawn at age a if and only if condition (6) prevails at age $a-1$ but not at age a. This yields the probability $\Pr_{t\,i}^{a}$ of an optimal withdrawal at age a conditionally on a renewal up to age a:

$$\Pr_{t\,i}^{a} = \frac{\Phi\left(Q_{t\,i}^{a}\right) - \Phi\left(Q_{t\,i}^{a-1}\right)}{1 - \Phi\left(Q_{t\,i}^{a-1}\right)} \tag{7}$$

where Φ denotes the cumulative density function of ε_i. In the terminology of duration models, $\Pr_{t\,i}^{a}$ is nothing else than the hazard rate characterising the econometric model of patent duration. The corresponding survival function is $1-\Phi\left(Q_{t\,i}^{a-1}\right)$. Let Ω_a denote the subset of patents renewed up to at least age a whatever their application date and let I_i^a be a variable that takes value 1 if patent $i \in \Omega_a$ is renewed at age a and value 0 otherwise. The log-likelihood of withdrawal *versus* renewal at age a for a patent i, conditional on the fact that we know that $i \in \Omega_a$, is given by:

$$L_i^a = \mathrm{I}_i^a \ln \Pr{}_{t\,i}^a + \left(1 - \mathrm{I}_i^a\right) \ln\left(1 - \Pr{}_{t\,i}^a\right) \tag{8}$$

Summing over all ages and all patents, we obtain the following log-likelihood

$$L_{tot} = \sum_{a=1}^{A} \sum_{i \in \Omega_a} L_i^a \tag{9}$$

Note that a same patent appears several times in (9) but at different ages. Estimates of parameters α_k ($k \in \{0, \cdots, K\}$) and of parameters of the probability distribution of ε are obtained by maximising (9) with respect to all these parameters. The advantage of estimating the discrete time duration model developed above rather than an *ad hoc* duration model of patents relies on its structural specification that directly provides estimates of all parameters required to assess patent value.

2.2.2. *Real option specification with heterogeneity in the depreciation rate*

The basic specification of patent renewal decisions presented so far is featured by a deterministic dynamics of the rent. In such a framework, a patent owner is indifferent in choosing between a mechanism based on renewal fees or a mechanism based on a menu of patent duration and upfront fees corresponding to the discounted sum of renewal fees. In other words, commitment to a predefined withdrawal date is costless because no additional information is supposed to be revealed as the patent ages.

This is no longer true if the time path of the rent can differ *ex post* from the time path expected *ex ante*. Practitioners stress the importance of this type of uncertainty. An extra value then accrues from the renewal mechanism compared to a commitment to a predefined withdrawal age because it makes it possible to adapt the optimal decision to unforeseen shocks that modify the time path of the rent. This extra value is the value of flexibility that is typically accounted for in real option models (Dixit and Pindyck, 1994; Trigeorgis, 1996). Uncertainty affecting the dynamics of the rent makes the decay factor that is applied to obtain the rent at age a from its value at age $a-1$ in (3) specific to each patent. Both observed and unobserved heterogeneity in this decay factor are considered.

Observed heterogeneity is associated to the time path of observed variables that affect the depreciation of the rent. These variables may either be sector-based or macro-economic indicators of market conditions for patented inventions or patent specific characteristics like forward citations and patent litigation. The main point here is that their values change with the age of the patent. In reference to this property, such variables are referred to as dynamic variables. Let $Z_i^{t+a} = \left\{z_{1i}^{t+a}, \cdots, z_{mi}^{t+a}, \cdots, z_{Mi}^{t+a}\right\}$ denote the vector of values taken by the M dynamic variables z_{mi}^{t+a} ($m \in \{1, \cdots, M\}$) affecting at age a the depreciation of the rent for patent i applied for at date t. This vector conditions the depreciation rate g_{ti}^{a} associated to the observed component of the decay rate. In order to be consistent with the fact that this depreciation rate ranges between 0 and 1, a logistic specification is more specifically convenient:

$$g_{ti}^{a} = \frac{1}{1 + \exp\left(\beta_0 + \sum_{m=1}^{M} \beta_m \, z_{mi}^{t+a}\right)} \tag{10}$$

where β_m (with $m \in \{0, \cdots, M\}$) are parameters to be estimated. Unobserved heterogeneity is associated to factors affecting the depreciation of the rent and that are known to the patent owner but not to other economic agents. “Good” or “bad” information about technological opportunities offered by the patented invention typically belongs to this category. As a result, unobserved heterogeneity is captured by idiosyncrasic random terms. It is more precisely assumed that, between two renewal dates, there is a series of N random events that reduce the rent in a multiplicative form. Observed and unobserved heterogeneity are also assumed to interact multiplicatively for a technical reason that will be made explicit latter on. Accordingly, the decay factor of the rent between age $a-1$ and age a may be written as

$$\delta_{t\,i}^{a} = \left(1 - g_{t\,i}^{a}\right)\prod_{n=1}^{N}\left(1 - \tilde{\theta}_{n\,i}^{a}\right) \tag{11}$$

The depreciation rates $\tilde{\theta}_{n\,i}^{a}$ ($n \in \{1, \cdots, N\}$ and $a \in \{0, \cdots, A\}$) associated to the random events affecting the rent for each patent i are assumed to be identically and independently distributed in the range $[0\,,1]$ so that Assumption 1, Proposition 1 and Corollary 1 are satisfied. Uncertainty about the future time path of the rent arises from events underlying both unobserved and observed heterogeneity. Nevertheless, patent owners can make rational expectations about future values of $\delta_{t\,i}^{a}$ conditional on the information avalaible to them at the current time. “Good” news (respectively “bad” news) are then defined as realisations of the $\tilde{\theta}_{n\,i}^{a}$ and $z_{m\,i}^{t+a}$ that generate higher (respectively lower) than initially expected values of $\delta_{t\,i}^{a}$. Substituting (11) in (3) and taking the natural logarithm we obtain that

$$\ln R_{t\,i}^{a} = \ln R_{t\,i}^{0} + \sum_{s=1}^{a}\ln\left(1 + g_{t\,i}^{a}\right) + \sum_{s=1}^{a}\sum_{n=1}^{N}\ln\left(1 - \tilde{\theta}_{n\,i}^{s}\right) \tag{12}$$

Combining with (4) yields

$$\ln R_{t\,i}^{a} = \ln \alpha_0 + \sum_{k=1}^{K} \alpha_0 \ln x_{k\,i} + \sum_{s=1}^{a} \ln\left(1 + g_{t\,i}^{a}\right) + \ln \varepsilon_i + \sum_{s=1}^{a} \sum_{n=1}^{N} \ln\left(1 + \tilde{\theta}_{n\,i}^{s}\right) \tag{13}$$

Condition (6) for a patent i applied for at date t to be still alive at age a then becomes

$$\ln \varepsilon_i + \sum_{s=1}^{a} \sum_{n=1}^{N} \ln\left(1 + \tilde{\theta}_{n\,i}^{s}\right) > \ln Q_{t\,i}^{a-1} \tag{14}$$

with $Q_{t\,i}^{a-1}$ already defined in (6) except that $1 - g_{t\,i}^{a}$ has to be substituted to δ_t^a.

The next step in specifying the econometric duration model consists in obtaining the probability distribution of the left hand side of (14). At this stage, the assumption of a multiplicative interaction between random events affecting the unobserved component of depreciation is useful. Indeed, for a sufficiently high number N of i.i.d. random events between two successive renewal dates, the central limit theorem applies and (14) may be rewritten as

$$\ln \varepsilon_i + \tilde{\omega}_i^{a} > \ln Q_{t\,i}^{a-1} \tag{15}$$

Where $\tilde{\omega}_i^{a}$ is a Gaussian random term with a negative expected value $-\mu a$ and variance $\upsilon^2 a$. Parameters $\mu > 0$ and $\upsilon > 0$ correspond respectively to the opposite of the expected value and to the standard deviation of the unobserved component $\sum_{n=1}^{N} \ln\left(1 + \tilde{\theta}_{n\,i}^{s}\right)$ of the decay factor at age a. In order to proceed to the computation of the hazard rate of the discrete time model of patent duration as in (7), it is necessary to determine the cumulative probability distribution Φ of the sum of the two random variables that appear in the left hand side of (15). An immediate solution is to postulate a normal distribution for $\ln \varepsilon$ (with expected value 0 and variance σ^2) so that the probability distribution of the sum of the two terms in the left hand side of (15) is a normal probability distribution with expected value $-\mu a$ and variance $\sigma^2 + \upsilon^2 a$.

III. DATA COLLECTION AND VARIABLES

The dataset used in this paper consists of all EPO patents designating France and applied for in 1989. This choice relies on the fact that European patents designating France have been regularly studied in the economic literature (Schankerman and Pakes, 1986) thus allowing comparisons and has for advantage that contrary to some countries, renewal fees have been used for a long time and since the first age of the patent in France. Our comprehensive dataset of 26 904 patents was courteously provided by *Qwestel*®. Table 1 details the frequencies of withdrawals for the eight main technological fields in the International Patent Classification (IPC)[5]. The official code and description of each technological field are reported in Table 1. The profiles of withdrawal frequencies are similar across technological fields, except in the case of technological field E ("*fixed constructions*") for which more patents were withdrawn earlier. Table 1 also displays the average renewal fees to be paid at the different ages of a patent.

Insert Table 1

Existing automated scorings that aim at gauging the overall quality of patents try to provide a yardstick for measuring and comparing patent value. The premise of these rating

[5] For a comprehensive description of IPC, consult the website of the World International Patent Office at http://www.wipo.int/classifications/en/.

models is that combining a number of predictor variables (also called metrics) revealed by the patent document itself helps identifying patents that are statistically either more likely or less likely to produce economic returns. A brief description of the predictor factors used in our model is provided below. Two kinds of predictor variables have been distinguished and used to measure patent quality. On the one hand, static variables that have an impact on the initial rent but not on the dynamics of the rent. On the other hand, dynamic variables, i.e. variables that conversely have an impact on the dynamics of the rent and which, as such, play a crucial role when it comes to reassess the patent over time.

Static Variables (or time-invariant variables) :

- *Size* : number of patents from the same applicant recorded in the database. The underlying idea is to capture the size or the innovative intensity of applicants.
- *nbIPC:* number of IPC subclasses (at the 4-digit IPC level) as declared in the patent application. This variable measures whether the patent is broad or narrow. If numerous subclasses are applied for by the applicant, one may expect diffusion of the innovation in different domains.
- *speIPC:* percent of declared IPC subclasses that belong to the main IPC class of the patent. This variable measures whether what is declared as the main IPC class of the patent (the one that concentrates the highest number of subclasses declared by the patent) largely dominates other IPC classes or not. This variable can be interpreted as a measure of patent specialisation.
- *entropyIPC:* entropy of declared IPC classes (measured on the basis of the eight main IPC classes). This variable completes speIPC with the advantage that it takes into account the fact that declared IPC classes may concentrate on some classes but with the disadvantage that only the eight main IPC codes are used.

- *Claims*: number of independent and dependent claims in logarithm. The underlying idea is that the more claims a patent has, the broader the likely scope of protection and the better the likelihood of surviving a validity trial.

- *Keywords:* number of words in some key sentences in logarithm (information on this specific metric was provided by Qwestel®).

- *Ncte :* number of backwards citations. For EP publications, this field also contains opposition citations (reasons for opposition) and observer citations (i.e. examiner references). This variable describes the scope of prior art and signals valuable technological knowledge (De Carolis, 2003).

- *Ncta:* number of backwards citations by the applicant.

- *Npr:* number of recorded priority claims, in logarithm. Intuitively, more priority claims probably means a patent is entitled to an earlier filing date, which can be beneficial in fending off art-based validity attacks. It can also indicate a greater level of overall interest and investment by the patentee.

- *Npn:* family size, in logarithm. Lanjouw and al. (2004) found that family size is highly correlated with other indicators of patent value.

- *Ncc:* number of countries in the patent family, in logarithm.

- *Pct:* dummy variable taking value one if and only if the patent is applied for via the PCT procedure.

- *Ndsep:* number of European countries in which the patent is taken out, in logarithm. The geographical scope of the patents reveals the expectations of the patent applicant concerning patent value (Reitzig, 2004).

- *Univ*: dummy variable related to patent ownership and taking value one if and only if the applicant is an academic institution. This variable is not available for technological fields D *(“Textiles; Paper*”), E (*“Fixed constructions*”) and F (*“Mechanical engineering; Lightening; Heating; Weapons; Blasting”*) because too few patents were coded one with the result that the dummy could capture idiosyncratic shocks rather than the effect of ownership.

- *Firm&Univ*: dummy variable taking value one if and only if the applicant is a firm associated to an academic institution. Again, this variable is not available for technological fields D, E and F.

Insert Table 2

Some descriptive statistics on static variables are displayed in Table 2. No major differences appear between technological fields as regards these variables. However, similarities as regards the mean of static variables may hide different distributions of these variables. As it is not expected that parameter estimates for renewal models are systematically similar across technological fields, the model described in the previous section is estimated separately for each technological field.

Dynamic Variables (or time-dependent variables) :

- *Age:* age of the patent. This variable captures the fact that *ceteris paribus* the depreciation rate of the rent may be greater in the early ages.

- *Gdp:* GDP growth rate of the current year. This variable captures the general health of the economy.

- *Fcit:* forward citations rate, i.e. number of new citations received at the different ages of the patent. Some studies[6] have suggested that the number of citations or references made to an issued patent by other subsequently issued patents (forward citations) may have a positive correlation with economic value. Intuitively, a high forward citation rate could indicate a high level of commercial interest or activity in the patented technology.

<u>Insert Figure 2</u>

Figure 2 shows the average profile of forward citations received at each age as a function of patent age for the different technological fields. Forward citations are mainly received between age four and fifteen in all technological fields. The maximum average number of forward citations is received early, at around age four or five and generally, it slightly decreases up to age fifteen. A more drastic drop is then observed. Technological fields A (*"Human necessities*"), C ("*Chemistry; Metallurgy"*) and D (*"Textiles; Paper"*) on the one hand, and B ("*performing operations; transporting*"), F ("*Mechanical engineering; Lightening; Heating; Weapons; Blasting*") and E ("*Fixed constructions*") on the other hand respectively exhibit the highest and the lowest average number of citations. The average number of new citations for technological field A is about twice that of field E at most ages.

4. RESULTS

4.1 ESTIMATION RESULTS

Maximisation of the likelihood function (9) with the stochastic specification (15) of the probability of an optimal withdrawal at a given age conditional on renewal up to that age has

[6] Harhoff, D. & al. (1997).

been implemented to the dataset described in the previous section. The model has been estimated separately for each IPC technological field.

Insert Figure 3

Figure 3 provides some general insights into how the model fits the data. For this purpose, it follows a common practice that consists in comparing observed and simulated cumulated frequencies of withdrawals for a same cohort of patents at different ages. Three different methods are used to generate simulated frequencies of withdrawals.

The first method consists in generating one thousand random draws per patent of the initial value of the rent and its time path. Simulations for the time path of the rent are based on additional assumptions as regards dynamic variables which exact values are treated as unknown at the beginning of the patent life. The annual growth rate of the economy is assumed to follow a Gaussian probability distribution and it is supposed that the stochastic process of forward citations obeys a Poisson process[7]. The withdrawal age is computed for each random draw and the optimal withdrawal age finally forecasted for a patent represents the average of these different ages. The use of numerous random draws per patent minimises the role played by rare events in differentiating the dynamics of the rents. As a result, the first method may be thought of as a method that tends to neutralise unexplained important factors affecting the decision to renew or not a patent. On the one hand, this method makes sense to forecast patent duration at the beginning of their life. On the other hand, it induces very little

[7] More precisely, forward citations received at age a by a patent are modelled as the outcome of a Poisson count data model which parameter is a function of the cumulated forward citations received by the patent up to age $a-1$. Such a Poisson model has been estimated age by age. The fact of conditioning on the cumulative citations received turns out to have a statistically highly significant impact.. Detailed estimation results are available from the authors upon request.

variations in the forecasted optimal age of withdrawal so that, as shown by Figure 3, the frequencies of withdrawal for the cohort are much more concentrated than observed frequencies, whatever the technological field considered.

The second and third methods attempt to provide further evidence of this result. The second method is similar to the first one with the noticeable exception that only one initial value and one time path of the rent are generated for each patent. Therefore, the role of rare events is emphasised, some patents benefiting from important unexplained positive shocks while the others suffer from important negative unexplained shocks. The resulting cumulated frequency of withdrawal for the cohort moves closer to the one that is observed compared to the first method. This result argues in favour of the role of unexplained important shocks. Nevertheless, simulated frequencies of withdrawal remain much more concentrated than the observed ones.

The third method thus goes one step further by replacing the series of simulated forward citations and simulated economic growth rates by the observed ones. As a consequence, the simulated cumulated frequency of withdrawal for the cohort overlaps almost perfectly with the one observed whatever the technological field considered. This third method thus outlines the importance of correctly forecasting the time path of dynamic variables which requires a more complex modelling than what was postulated in the first two methods.

Insert Figure 4

In order to give a general idea of the estimated initial value and dynamics of the rent, Figure 4 shows the mean and median time paths of the rent that are obtained with the third method already used to generate Figure 3[8]. For each technological field, these time paths are compared to the average profile of renewal fees on the period covered by the dataset. In order

[8] Broadly speaking, results obtained with the first and second methods do not drastically differ.

to facilitate the comparison between initially high rents and low renewal fees, both are expressed in natural logarithm. A first common feature of all technological fields is that the mean time path always exceeds the median time path, thus suggesting an asymmetry in the distribution of rents with some very high values whatever the age considered. The absolute gap between the natural logarithm of the mean and median time paths is approximately constant over all ages. Consequently, the ratio of the mean to the median time path is also constant, indicating that the degree of asymmetry does not change drastically as patents age. A second common feature is that the mean time path of the rent is never below the annual fees thus justifying maintaining the patent in force up to the statutory life limit. By contrast, the median time path falls below the renewal fee at around age thirteen for all technological fields.

Figure 4 also highlights some differences between technological fields. Technological field D ("*Textiles and Paper*") appears to be the most atypical one with a high initial mean but median values of the rent. A higher depreciation rate of the rents largely counterbalances the higher initial values of the rents so that, for instance, the median time path does not pass below annual fees at a later age compared to what is observed for other technological fields. The high gap between mean and median time paths indicates that the asymmetry between patents with the highest values and those (the majority) with middle or low values is important. To a lesser extent, technological field H ("*Electricity*") exhibits a similar profile in terms of the dynamics of the rents. This atypical profile suggests that patents protect strategically important and independent inventions but that the pace of innovation is faster in these two technological fields compared to other technological fields. The profile of the dynamics of the rents for technological fields E ("*Fixed constructions*") F ("*Mechanical engineering; Lighting; Heating; Weapons; Blasting*") and B ("*Performing operations; Transporting*") is just opposite to the profile of technological fields D and F. Initial values of the rents are low and not highly different (as indicated by the low gap between the mean and median) but are counterbalanced by a slower depreciation rate. This is typical of technological fields where incremental and highly complementary innovations dominate. The other technological fields have intermediate profiles in terms of the dynamics of rents.

Insert Table 3

Table 3 reports indicators related to the global quality of estimates. The percentage of total variance of the natural logarithm of the initial rent explained by the model is used to assess whether patent heterogeneity in terms of the initial rent is correctly captured by the static variables of the model. The log linearity of the expression of the initial rent makes its computation easy. Similarly, the log linearity of expression (13) facilitates the computation of the percentage of total variance of the natural logarithm of depreciation up to a given age that is explained by the dynamic variables of the model. Though relatively standard when dealing with micro level data, these two percentages are clearly too low to enable a reliable forecasting of patent duration. This is confirmed by the comparison between the average probability of withdrawal at age ten for patents effectively withdrawn at this age and for patents renewed at this age. Patents still alive at age ten only have been used to compute the two average probabilities. The focus on the midterm of the maximum patent duration offers an interesting compromise between the necessity to take into account the realisations of the dynamic variables of the model on a sufficiently long period and the requirement of a sufficiently large subset of patents still alive for results to be statistically reliable. The difference between the two average probabilities has the expected sign but is very low. More importantly, the probability of withdrawal for patents effectively withdrawn never exceeds 0.5 so that a prediction of withdrawals on the basis of this probability can hardly be performed. This is not in contradiction with the almost perfect overlap of the observed cumulated frequencies of withdrawal and the simulated cumulated frequencies obtained with the third simulation method in Figure 3. It just reveals that, due to the role of unobserved heterogeneity, the profile of withdrawals at the level of the cohort is correctly predicted but withdrawals at the patent level are not.

This result is important because it casts some doubts about the possibility to implement a reliable patent scoring with some mitigation depending on the technological field considered. Technological fields A ("*Human necessities*") and H ("*Electricity*") for instance are the two technological fields for which the initial rent is the less correctly explained by the model but, by contrast, the dynamics of the rent is better explained than for all the other technological fields except technological field D ("*Textiles; Paper*"). A patent scoring at a too early age would thus probably be affected by important errors but results could be improved as time passes and that sufficient information is revealed about the realisation of dynamic variables that could then allow discriminating among patents. At the opposite, the best explanation for the initial rent is for technological field E ("*Fixed constructions*") but at the expense of the dynamics of the rent which is the worst correctly explained. This leads us to conclude that there is little to be expected from improving patent scoring by using patent age and new information. The technological fields exhibiting the best compromise between a correct explanation of the initial rent, on the one hand and a correct explanation of its dynamics, on the other hand are technological fields B ("*Performing operations; Transporting*") and G ("*Physics*"). More details about the impact of static variables and dynamic variables on respectively the initial rent and its dynamics are provided in the two following tables.

Insert Table 4

Table 4 reports coefficients of static variables influencing the expected natural logarithm of the initial rent. When focusing on coefficients statistically significant at a 10% confidence level, it appears that each static variable generally impacts the initial rent in the same direction, whatever the technological field. The two exceptions are variables *Size* and *speIPC*. Among the four technological fields for which *Size* has a significant impact on the initial rent, two exhibit a positive impact (B (“*Performing operations; Transporting*”) and G (“*Physics*”)) and the two others (C (“*Chemistry; Metallurgy*”) and D (“*Textile; Paper*”)) are characterised by a negative impact. Thus, no general conclusions can be drawn as regards the innovative capacity of applicants on the value of their patents. This impact varies from one technological field to the other. The variable *speIPC* has a positive impact for technological fields A (“*Human necessities*”), G (“*Physics*”) and H (“*Electricity*”) and a negative impact for technological field D (“*Textiles; Paper*”). The degree of specificity versus generality (as measured by the percentage of declared IPC subclasses that belong to the class of interest) is neutral for all other technological fields. The static variable that influences the highest number of technological fields in a concordant positive direction, except technological field D (“*Textiles; Paper*”), is the number of words in key sentences (*keywords*). It is followed by the number of claims (*Claims*). Other variables having a concordant and significant positive impact, although for a lower number of technological fields, are the number of IPC classes targeted by the patent (*nbIPC*), the number of backward citations made by the examiner (*Ncte*), the family size (*Npn*), the fact that the patent is applied for via the PCT procedure (*Pct*) and the number of European countries targeted by the patent (*Ndsep*). The other static variables have no significant influence on the initial rent.

Another way to read estimation results reported in Table 4 consists in looking at those technological fields for which the number of significant metrics is the highest. Technological field C (“*Chemistry; Metallurgy*”) clearly emerges as the technological field with the highest number of significant coefficients followed by technological fields A (“*Human necessities*”), B (“*Performing operations; Transporting*”), G (“*Physics*”) and H (“*Electricity*”). Note that these technological fields are not necessarily those with the highest percentage of total variance in the initial rent explained by the model.

<u>Insert Table 5</u>

Estimated values of coefficients related to observed variables that affect the dynamics of the rent are displayed in the upper part of Table 5. A striking result is that most of these coefficients are highly significant and modify the depreciation rates of the rent in the same way whatever the technological field considered. A positive sign of the coefficient associated to a dynamic variable means that the depreciation rate decreases with the variable (see expression (10)). Accordingly, the depreciation rate is lower *ceteris paribus* in the initial period of a patent life than when it is close to its statutory life limit (variable *Age*) for technological field B ("*Performing operations; Transporting*") and F ("*Mechanical engineering; Lighting; Heating; Weapons; blasting*") .

Similarly, new forward citations (*Fcit*) received by a patent decrease its depreciation rate at the date these citations are received and thus yield higher values of the rent at all subsequent dates for all technological fields except technological field G ("*Physics*"). Finally, the depreciation rates of patents are systematically and negatively correlated to the GDP growth rate (variable *Gdp*). Indeed, the GDP growth rate of the current year always has a positive coefficient in Table 5.

The lower part of Table 5 helps understanding the net impact of dynamic variables on the depreciation rate of the rent. Average values of the estimated deterministic component of annual depreciation rates at age one and ten are first displayed. The fact that most patents are still alive at age ten and that, at the same time, a sufficient number of forward citations have been received at this age to generate heterogeneity in the evolution of the rents justifies the focus on this age. The comparison between age one and age ten clearly shows that, in absolute terms, the deterministic component of annual depreciation rates increases. Thus, the accumulation of forward citations (variable *Fcit*) and economic growth (variable *Gdp*) do not counterbalance a natural tendency of depreciation rates to accelerate (variable *Age*). The average sensitivity to changes in dynamic variables at age ten is quite similar from one technological field to another one and for an additional citation on the one hand or a one percent more economic growth on the other hand. The estimated average drop for an additional citation ranges between -0.08 for technological field D ("*Textiles; Paper*") and -0.14 for technological field H ("*Electricity*"). These figures mean that if the depreciation rate was for instance 40% before the new citation is received, then it falls respectively at 32% or 26% after the new citation is received. As a result, patents receiving many forward citations may have a much lower depreciation rate than other patents. Technological fields that are the most sensitive to a one percent GDP increase are not systematically the same than those that are the most sensitive to the receipt of a new forward citation. Indeed, the estimated average drop of the depreciation rate associated to an additional point of economic growth ranges from -0.08 for technological field F ("*Mechanical engineering; Lighting; Heating; Weapons; Blasting*") up to -0.17 for technological field H ("*Electricity*").

Whether unobserved heterogeneity has a significant impact on the dynamics of the rents cannot be evaluated on the basis of t-statistics for two reasons. The first reason is that the significance of the two associated coefficients cannot be tested separately. The second reason is that coefficients reported in Table 5 are not directly the expected value and standard deviation of random shocks but their natural logarithm[9]. Consequently, the role of unobserved heterogeneity is rather assessed on the basis of a log-likelihood ratio. This is why Table 3

[9] Instead of being specified as $\mu > 0$ and $\upsilon > 0$ the expected value and standard deviation are specified as $\exp(\mu')$ and $\exp(\upsilon')$ to make sure that they will have positive values.

reports the log-likelihood of the model with its dynamics restricted to observed heterogeneity and the chi square statistic to implement the test[10]. Unobserved heterogeneity plays a significant role in all technological fields except technological field D ("*Textiles; Paper*") and eventually technological field E ("*Fixed constructions*") if a low risk of error is imposed.

Note that even if the unobserved component of the dynamic is not statistically significant, the existence of a significant impact for one or several dynamic variables is sufficient to generate a patent option value. In this respect, the numerous and highly significant effects identified in Table 5, both for the observed and unobserved components of the dynamics, strongly advocate in favour of the existence of a patent option value whatever the technological field considered. Option value follows on from the important role played by the revealing of additional information as patents age. It thus justify the use of a renewal mechanism that provides greater flexibility in the withdrawal decision compared to a mechanism based on a upfront fee modulated in accordance to a predetermined withdrawal date. Meanwhile, the existence of an option value undermines the accuracy of any scoring system, at least at the early ages of a patent, due to the uncertainty that intrinsically affects future realisations of the rents.

4.2 SCORING RESULTS

As stressed by Barney (2002), although patent metrics with a significant coefficient in Table 4 and Table 5 are interesting and informative, individually they provide only limited guidance in determining overall patent quality. What we need to obtain is a single rating to be used to directly forecast or estimate the value of each patent. In the methodology used by *Ocean Tomo*®, patents are positioned relative to each other. Raw scores are first produced on the

[10] More precisely coefficients μ' and υ' have been restricted to -100 so that $\exp(\mu')$ and $\exp(\upsilon')$ are almost equal to zero.

basis of the estimated probability that each patent is maintained for the full statutory term. For convenience, raw scores are adjusted to provide a normalised mean or median score of 100. Thus, a score of 100 on the IPQ scale generally corresponds to an expected normal quality (average expected duration) while an IPQ score higher or lower than 100 indicates an expected above-average or below-average patent quality respectively. Of course, as with IQ, the IPQ score provides only part of the equation for determining patent quality/value. Thus, a high IPQ does not guarantee high quality/value and vice versa. It only establishes a statistical correlation based on the body of available data.

In our paper, we follow the same line of reasoning than *Ocean Tomo*® by positioning patents against each other. For this purpose, we compute the ratio of the estimated value of each patent to the median value times 100. However our methodology departs from that proposed by *Ocean Tomo*® as regards the basis for computing IPQ. Indeed, in accordance with the theoretical discussion on the link between patent renewals and patent value presented in section II, we compute our IPQ on the basis of an estimated monetary value for each patent rather than on the basis of maintenance rates. Monetary values are estimated by using one thousand random draws, conditional on observed static variables, of the initial rent and one thousand random draws of successive annual depreciation rates of the rent for each patent. The method used to generate these random draws is the same than the first method used to generate the cumulated frequencies of withdrawal in Figure 3. In order to stress the role of additional information acquired as patents age, IPQs have been computed at age one and ten. When computed at age one, IPQs are thus based on random simulations of cumulated forward citations and random simulations of GDP growth. Conversely, when computed at age ten, IPQs account for past realizations of dynamic variables that affect the probability distribution of the rent conditional on renewal up to age ten. Whatever the age considered, the use of numerous simulations for each patent enables us to generate an empirical distribution of IPQ for each patent and thus to determine the probability that a specific draw of the IPQ sharply departs from the expected IPQ. For each random draw, the optimal withdrawal age defined in Proposition 1 is determined and, then, the discounted sum of the rent net of renewal fees from the date of application to the optimal withdrawal date is computed[11]. The value affected to each patent is the average value obtained over the one thousand random draws.

Insert Figure 5

Figure 5 shows the empirical distribution of IPQs over all patents in a same technological field at age one and ten. Note that for graphical convenience, the distributions have been truncated at 800. Thus, the high frequencies of IPQs observed on the extreme right of each distribution only reflect that there is still an important mass of patents on the right tail of the distribution. The general shape of distribution does not sharply differ from one technological field to another and from year one to year ten. The distributions are systematically highly asymmetric with a mode associated to low values of the IPQ and a tail that spreads far on the right, up to more than 1000. This is in line with a well established result in the empirical literature. More interestingly, for most technological fields, the asymmetric distribution of IPQs is more pronounced at age ten than at age one. Indeed, the mode for low values of IPQs and for the majority of patents with an IPQs above 800 increases whereas the proportion of patents with intermediate IPQs slightly decreases. As a result, patents are more discriminated at age ten than at age one, a result that follows on from the additional information available to compute IPQs. Technological field G ("*Physics*") and to a less extent technological field D ("*Textiles; Paper*") are two exceptions. Technological field G ("*Physics"*) is more specifically

[11] The discount rate is set at 3%. Clearly, this choice is arbitrary and justifies working with a normalised score like IPQ rather than on estimated monetary values.

characterized by a decrease of patents with an IPQs higher than 800 at age ten than at age one, a characteristic which may be related to the sharp increase of the deterministic component of the depreciation rate reported in Table 5.

Insert Figure 6

Figure 6 is intended to assess whether IPQs provide a reliable scoring or not. For this purpose, empirical distributions of the monetary values associated to the one thousand random draws have been generated for the two patents with respectively the lowest and the highest IPQ in each technological field. Monetary values are normalized by the median value of patents in the technological field, times one hundred, to be consistent with the computation of an IPQ. Comparisons between the two distributions have been made for age one and age ten. Nevertheless, because the results obtained for age ten systematically correspond to 100% of IPQs at the extreme left for the patent with the lowest average IPQ and 100% at the extreme right for the patent with the highest average IPQ, they are not displayed in Figure 6. This polarization of IPQs reveals an unambiguous ranking of these two patents and constitutes a positive signal as regards the reliability of an IPQ scoring when applied to patents at mid term. Though less pronounced, this is also the case at age one for technological fields B ("*Performing operations; Transporting*"), E ("*Fixed constructions*"), F ("*Mechanical engineering; Lighting; Heating; Weapons; Blasting*") and G ("*Physics*"). Conversely, for technological fields A ("*Human necessities*"), C ("*Chemistry; Metallurgy*"), D ("*Textiles; Paper*") and H ("*Electricity*"), the patent with the highest average IPQ dominates the patent with the lowest IPQ only because of some abnormal IPQ draws that pull the average IPQ upwards. An IPQ scoring is thus not reliable at early ages for these technological fields.

V. CONCLUSION

A hallmark of a properly functioning marketplace for IPR is that there is a clear way to determine the fair price of the assets being bought and sold. It goes without saying that the creation of a rating scheme and of a robust and efficient financial market for intangibles should add considerable value. But this is challenging in itself and the task has been made even more difficult by the recent turmoil in the financial market and accusations that credit rating agencies (CRAs) are plagued by conflicts of interest that might inhibit them from providing accurate and honest ratings. Patent rating agencies are not exempt from such criticisms. This explains in part why European authorities and some large European companies are considering transparency standards for patent rating agencies and the obligation to publish their valuation methods. It is indeed important for market operators and investors to understand how a specific rating was determined and to have assessments of the uncertainty surrounding scoring results.

Our paper was aimed at studying the feasibility of such a rating scheme. We show that statistically derived patent performance benchmarks can provide objective measures of comparative patent quality and/or value but at the cost of potentially important uncertainties. Indeed, it is likely that some low or high scores could result from unexplained and random factors beyond the control of any rating agency. Moreover, it is worth keeping in mind that the existence of an option value undermines the accuracy of any scoring system, at least at the early ages of a patent, due to the uncertainty that intrinsically affects future realisations of the rents. Therefore, our results cast some doubt on the possibility to develop a reliable rating system based only on patent metrics.

TABLE 1.
STATISTICS ON PATENT RENEWALS

Age	A Human necessities	B Performing operations; Transporting	C Chemistry; Metallurgy	D Textiles; Paper	E Fixed constructions	F Mechanical engineering; Lighting; Heating; Weapons; Blasting	G Physics	H Electricity	Average renewal fee on the period 1989 to 2009 In Euros
	Sample size								
	3641	5374	6235	680	638	2204	4595	3537	
	Observed frequencies of withdrawal								
1	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	49.55
2	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	26.81
3	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	29.47
4	0.27%	0.19%	0.08%	0.29%	0.31%	0.23%	0.09%	0.08%	32.48
5	1.43%	1.30%	0.79%	2.21%	2.19%	2.00%	0.37%	0.59%	41.09
6	3.63%	5.08%	3.50%	3.24%	8.15%	6.03%	2.72%	1.41%	82.46
7	6.73%	7.15%	5.92%	4.12%	10.34%	7.67%	4.20%	4.44%	98.73
8	8.71%	8.49%	7.89%	8.09%	10.97%	7.17%	8.55%	7.72%	117.84
9	7.33%	7.13%	7.41%	8.24%	8.46%	6.94%	8.23%	8.37%	140.24
10	6.84%	6.74%	7.25%	7.50%	7.05%	6.58%	5.68%	6.64%	161.91
11	5.66%	6.59%	6.56%	5.29%	6.43%	6.90%	6.99%	6.87%	220.61
12	4.70%	5.38%	5.98%	6.91%	5.02%	5.76%	5.18%	5.80%	245.72
13	4.39%	5.10%	4.67%	4.85%	5.33%	6.08%	4.44%	4.89%	274.07
14	5.16%	5.95%	6.53%	5.74%	6.11%	5.85%	6.25%	6.93%	300.76
15	5.22%	5.32%	6.85%	8.09%	4.86%	5.76%	6.12%	7.35%	329.11
16	4.92%	4.69%	4.78%	4.26%	3.92%	5.13%	5.20%	5.63%	434.68
17	4.28%	3.67%	3.80%	3.82%	2.82%	4.17%	5.16%	4.66%	464.70
18	4.15%	4.24%	3.85%	5.00%	2.51%	4.17%	4.48%	4.07%	495.57
19	3.10%	4.06%	2.87%	4.85%	2.82%	2.99%	5.05%	4.18%	527.05
20	6.65%	6.29%	5.42%	6.62%	4.08%	4.76%	7.33%	6.93%	563.78
21	16.84%	12.65%	15.86%	10.88%	8.62%	11.80%	13.97%	13.43%	

TABLE 2.
DESCRIPTIVE STATISTICS FOR STATIC VARIABLES

	A	B	C	D	E	F	G	H
	Human necessities	Performing operations; Transporting	Chemistry; Metallurgy	Textiles; Paper	Fixed constructions	Mechanical engineering; Lighting; Heating; Weapons; Blasting	Physics	Electricity
Size	31.25 (68.11)	46.78 (86.56)	78.39 (101.98)	48.95 (80.56)	11.47 (37.20)	34.39 (67.27)	105.96 (127.47)	118.07 (133.48)
nbIPC	2,81 (1,50)	2,83 (1,71)	3,67 (1,91)	2,86 (1,71)	2,54 (1,48)	2,70 (1,40)	2,54 (1,51)	2,49 (1,40)
speIPC	59,55 (32,42)	52,39 (33,04)	58,12 (25,32)	42,40 (34,36)	60,44 (35,14)	50,33 (32,40)	48,31 (26,46)	48,17 (26,60)
entropyIPC	0,46 (0,38)	0,47 (0,41)	0,51 (0,37)	0,56 (0,39)	0,48 (0,43)	0,46 (0,42)	0,43 (0,40)	0,39 (0,41)
Claims	16,51 (17,59)	12,70 (9,40)	17,03 (17,39)	12,61 (9,12)	11,87 (8,05)	11,54 (8,24)	14,37 (12,44)	13,20 (11,51)
Keywords	463,72 (307,70)	512,47 (260,48)	440,05 (264,19)	462,42 (225,04)	494,82 (260,08)	519,71 (266,99)	533,84 (268,35)	527,07 (267,01)
Ncte	3,46 (2,54)	3,80 (2,47)	3,19 (2,29)	3,49 (2,14)	3,94 (2,85)	3,97 (2,66)	3,41 (2,74)	3,12 (2,21)
Ncta	0,03 (0,45)	0,02 (0,36)	0,05 (1,31)	0,01 (0,34)	0,02 (0,36)	0,00 (0,12)	0,05 (0,77)	0,01 (0,15)
Npr	2,27 (1,23)	2,11 (1,23)	2,18 (1,27)	1,94 (1,10)	2,25 (1,18)	2,04 (1,15)	1,94 (1,36)	1,81 (1,13)
Npn	9,31 (5,26)	7,27 (4,06)	8,43 (4,79)	7,73 (3,80)	7,57 (3,97)	6,86 (3,34)	6,29 (3,15)	6,20 (2,82)
Ncc	8,61 (4,70)	6,59 (3,36)	7,73 (4,26)	7,15 (3,37)	7,00 (3,74)	6,23 (2,88)	5,71 (2,59)	5,72 (2,52)
Pct	0,22 (0,41)	0,18 (0,38)	0,16 (0,37)	0,13 (0,34)	0,22 (0,42)	0,21 (0,41)	0,18 (0,38)	0,16 (0,36)
Ndsep	9,69 (3,38)	7,42 (3,51)	8,50 (3,50)	7,79 (3,20)	8,67 (3,40)	6,73 (3,34)	6,29 (3,39)	5,99 (3,23)
Univ	0,02 (0,14)	0,001 (0,05)	0,02 (0,13)	-	-	-	0,01 (0,08)	0,001 (0,06)
Firm&Univ	0,01 (0,08)	0,001 (0,03)	0,01 (0,08)	-	-	-	0,001 (0,04)	0,001 (0,04)

TABLE 3.
GLOBAL INDICATORS OF ESTIMATION RESULTS

A	B	C	D	E	F	G	H
Human necessities	Performing operations; Transporting	Chemistry; Metallurgy	Textiles; Paper	Fixed constructions	Mechanical engineering; Lighting; Heating; Weapons; Blasting	Physics	Electricity
Log likelihood of the complete model							
$-9.908*10^3$	$-1.484*10^4$	$-1.682*10^4$	$-1.889*10^3$	$-1.739*10^3$	$-6.109*10^3$	$-1.252*10^4$	$-9.586*10^3$
Log likelihood of the model with dynamics restricted to observed heterogeneity							
$-1.829*10^4$	$-3.802*10^4$	$-3.024*10^4$	$-2.434*10^3$	$-4.948*10^3$	$-1.790*10^4$	$-3.207*10^4$	$-1.787*10^4$
Chi square statistic for the log-likelihood ratio test of restriction to a non stochastic dynamics							
16.76	46.36	26.84	1.09	6.42	23.58	39.10	16.56
% of total variance of the natural logarithm of the initial rent explained by the model							
8.58	16.81	13.90	12.60	28.84	12.95	16.4579	9.9758
% of total variance of the natural logarithm of depreciation up to age 10 explained by the model							
28.11	14.76	30.59	19.01	10.26	11.79	20.36	29.43
Average probability of withdrawal at age 10 for patents effectively withdrawn at age 10							
0.0834	0.0929	0.0912	0.0821	0.1199	0.0978	0.0776	0.0821
Average probability of withdrawal at age 10 for patents renewed at age 10							
0.0812	0.0908	0.0871	0.0798	0.1177	0.0967	0.0757	0.0789

TABLE 4.
ESTIMATION RESULTS FOR THE INITIAL RENT

	A	B	C	D	E	F	G	H
	Human necessities	Performing operations; Transporting	Chemistry; Metallurgy	Textiles; Paper	Fixed constructions	Mechanical engineering; Lighting; Heating; Weapons; Blasting	Physics	Electricity
Estimated coefficients for static variables (t-stats are reported in brackets, grey cells correspond to coefficients statistically significant at a 10% confidence level)								
Intercept	21.2660 (13.7637)	14.2947 (6.7753)	22.5300 (30.9868)	30.7070 (5.6114)	8.1877 (1.9651)	14.0031 (3.8652)	17.7109 (8.5633)	23.1772 (3.3055)
Standard deviation	4.2664 (8.1151)	3.1006 (18.5660)	4.1235 (8.1571)	7.0814 (2.4529)	2.4735 (10.3058)	3.0686 (8.6849)	3.2839 (11.8823)	4.5986 (6.0801)
Size	0.2179 (1.4226)	0.4719 (4.2490)	-0.4197 (-6.9914)	-0.7686 (-1.7202)	-0.2164 (-0.5563)	-0.1139 (-0.5065)	0.1845 (1.7525)	-0.0660 (-0.5164)
nbIPC	0.2605 (0.1063)	0.3442 (0.9659)	0.5414 (1.8900)	3.1060 (2.3040)	-0.3009 (-0.3126)	0.5461 (0.8679)	0.3489 (0.8708)	0.5216 (0.8990)
speIPC	0.0239 (2.7853)	0.0103 (1.3752)	-0.0056 (-0.9513)	-0.0625 (-2.3218)	-0.0074 (-0.4109)	-0.0016 (-0.1302)	0.0185 (2.4096)	0.0250 (2.5956)
entropyIPC	-0.2679 (-0.3561)	0.0281 (0.0521)	-0.4085 (-0.9643)	-1.7845 (-0.9373)	0.4279 (0.2940)	0.1045 (0.1208)	-0.4487 (-0.7799)	-0.4349 (-0.5668)
Claims	0.3044 (1.1430)	0.5439 (2.3710)	0.5151 (2.8818)	-0.8908 (-1.1747)	0.3182 (0.5766)	0.2707 (0.6832)	1.0703 (4.6271)	0.7117 (2.5632)
Keywords	0.4911 (1.5938)	1.0035 (3.4193)	0.5497 (2.7247)	0.7870 (0.7786)	1.7319 (2.6471)	1.3143 (2.3976)	0.8952 (2.3709)	1.1054 (2.6823)
Ncte	0.0316 (0.3913)	0.2846 (4.2616)	0.1135 (1.8363)	0.0656 (0.2497)	0.0684 (0.4427)	0.0671 (0.5526)	0.2155 (2.8401)	0.1445 (1.4161)
Ncta	0.1214 (0.2554)	0.3864 (0.6535)	0.0511 (0.4731)	-1.1308 (-0.9272)	0.0928 (0.0727)	1.2073 (0.3228)	0.2384 (0.7369)	0.3364 (0.1573)
Npr	-0.4312 (-0.8684)	0.0787 (0.1890)	-0.1819 (-0.5684)	-0.9561 (-0.6792)	-0.1776 (-0.1837)	-0.1495 (-0.1748)	0.3853 (0.8092)	0.2807 (0.4678)
Npn	3.2593 (1.9364)	1.1399 (0.8675)	2.9632 (2.6307)	0.5839 (0.1373)	1.6112 (0.5259)	2.1280 (0.8283)	1.8175 (1.3544)	3.2886 (1.7522)
Ncc	-1.8027 (-1.0729)	0.1704 (0.1284)	-1.4140 (-1.2610)	1.9939 (0.4548)	-0.1067 (-0.0373)	-0.9210 (-0.3544)	-1.4599 (-1.0263)	-2.2548 (-1.0153)
Pct	1.4424 (2.5790)	0.4472 (0.8691)	1.4454 (3.3925)	2.3339 (1.2277)	0.9695 (0.8586)	0.5031 (0.5385)	0.2262 (0.4019)	0.0740 (0.0997)
Ndsep	-0.3966 (-0.7281)	-0.1786 (-0.4673)	0.0245 (0.0751)	0.3424 (0.2463)	1.3958 (1.4766)	0.5125 (0.7207)	-0.0493 (-0.1190)	0.0056 (0.0102)
Univ	0.3394 (0.6643)	-1.3382 (-0.4696)	0.4550 (0.4202)	-	-	-	-1.1262 (-0.5638)	-2.0030 (-0.2930)
Firm&Univ	1.4594 (0.9366)	0.6355 (0.1224)	1.0101 (0.5851)	-	-	-	-0.1322 (-0.0413)	-0.7455 (-0.1498)

TABLE 5.
ESTIMATION RESULTS FOR THE DYNAMICS OF THE RENT

	A	B	C	D	E	F	G	H
	Human necessities	Performing operations; Transporting	Chemistry; Metallurgy	Textiles; Paper	Fixed constructions	Mechanical engineering; Lighting; Heating; Weapons; Blasting	Physics	Electricity
Estimated coefficients for the observed component of the depreciation rate (t-stats are reported in brackets, grey cells correspond to coefficients statistically significant at a 10% confidence level)								
Intercept	-2.1056 (-6.1505)	-1.2277 (-5.8046)	-2.2855 (-8.6828)	-2.7270 (-2.3040)	-1.7116 (-3.0174)	-0.7164 (1.6511)	-1.5316 (-1.3642)	-2.6499 (-4.176)
Age	-0.0215 (-0.5419)	-0.0978 (-3.3161)	0.0174 (0.8223)	-0.0185 (-0.1942)	-0.0495 (-0.5740)	-0.1633 (-2.9322)	-0.0912 (-1.3722)	-0.0243 (-0.5452)
Gdp	0.7236 (6.5277)	0.6016 (8.2406)	0.6136 (5.8644)	0.6556 (6.5017)	0.6191 (11.4521)	0.3951 (7.5249)	0.7563 (2.1886)	0.7262 (3.7744)
Fcit	0.4829 (3.4291)	0.5809 (4.6599)	0.4999 (8.5962)	0.3495 (2.5960)	0.5531 (7.8719)	0.6134 (7.3627)	0.6299 (1.4277)	0.6033 (2.9220)
Estimated coefficients for the unobserved component of the depreciation rate (t-stats are reported in brackets)								
Expected value	-0.2768 (-0.6849)	-0.3470 (-1.2870)	-0.3365 (-0.9429)	-0.2834 (-0.3469)	-0.3264 (-0.6579)	-0.7302 (-1.0071)	-0.3290 (-0.2501)	-0.2623 (-0.3806)
Standard deviation	-0.4233 (-2.3326)	-0.2388 (-2.2897)	-0.6279 (-4.8544)	-0.6763 (-0.8986)	-0.3512 (-0.8057)	-0.0286 (-0.1569)	-0.3398 (-1.6054)	-0.4309 (-2.060)
Average value of the deterministic component of depreciation rates at age 1 (standard deviations are reported in brackets)								
	0.2909 (0.0101)	0.2346 (0.0074)	0.4268 (0.0169)	0.5031 (0.0081)	0.3059 (0.0071)	0.3165 (0.0110)	0.1775 (0.0099)	0.4119 (0.0160)
Average value of the deterministic component of depreciation rates at age 10 (standard deviations are reported in brackets)								
	0.3960 (0.0996)	0.4825 (0.0928)	0.4448 (0.0953)	0.6182 (0.0714)	0.4849 (0.0738)	0.6889 (0.0938)	0.4017 (0.0950)	0.5335 (0.1094)
Average impact on the deterministic component of depreciation rates at age 10 (standard deviations are reported in brackets)								
One forward citation more	-0.1045 (0.0216)	-0.1358 (0.0186)	-0.1143 (0.0187)	-0.0833 (0.0030)	-0.1313 (0.0148)	-0.1380 (0.0096)	-0.1345 (0.0260)	-0.1427 (0.0192)
1% more of GDP growth rate	-0.1504 (0.0322)	-0.1404 (0.0193)	-0.1383 (0.0232)	-0.1583 (0.0069)	-0.1460 (0.0167)	-0.0866 (0.0063)	-0.1580 (0.0312)	-0.1706 (0.0239)

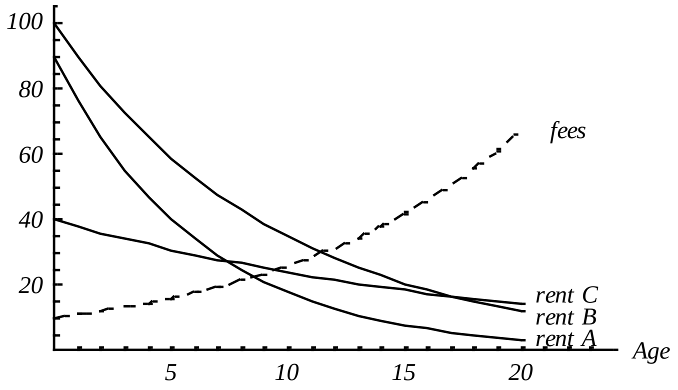

FIGURE 1. A SIMPLE NUMERICAL EXAMPLE

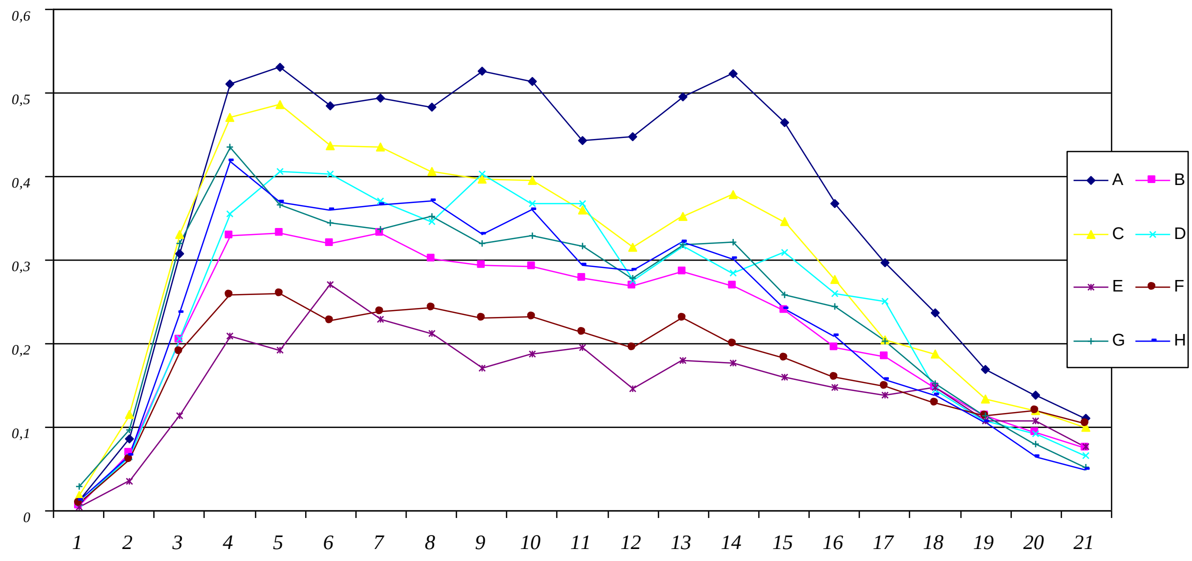

FIGURE 2. AVERAGE NUMBER OF RECEIVED FORWARD CITATIONS PER AGE AND TECHNOLOGICAL FIELD

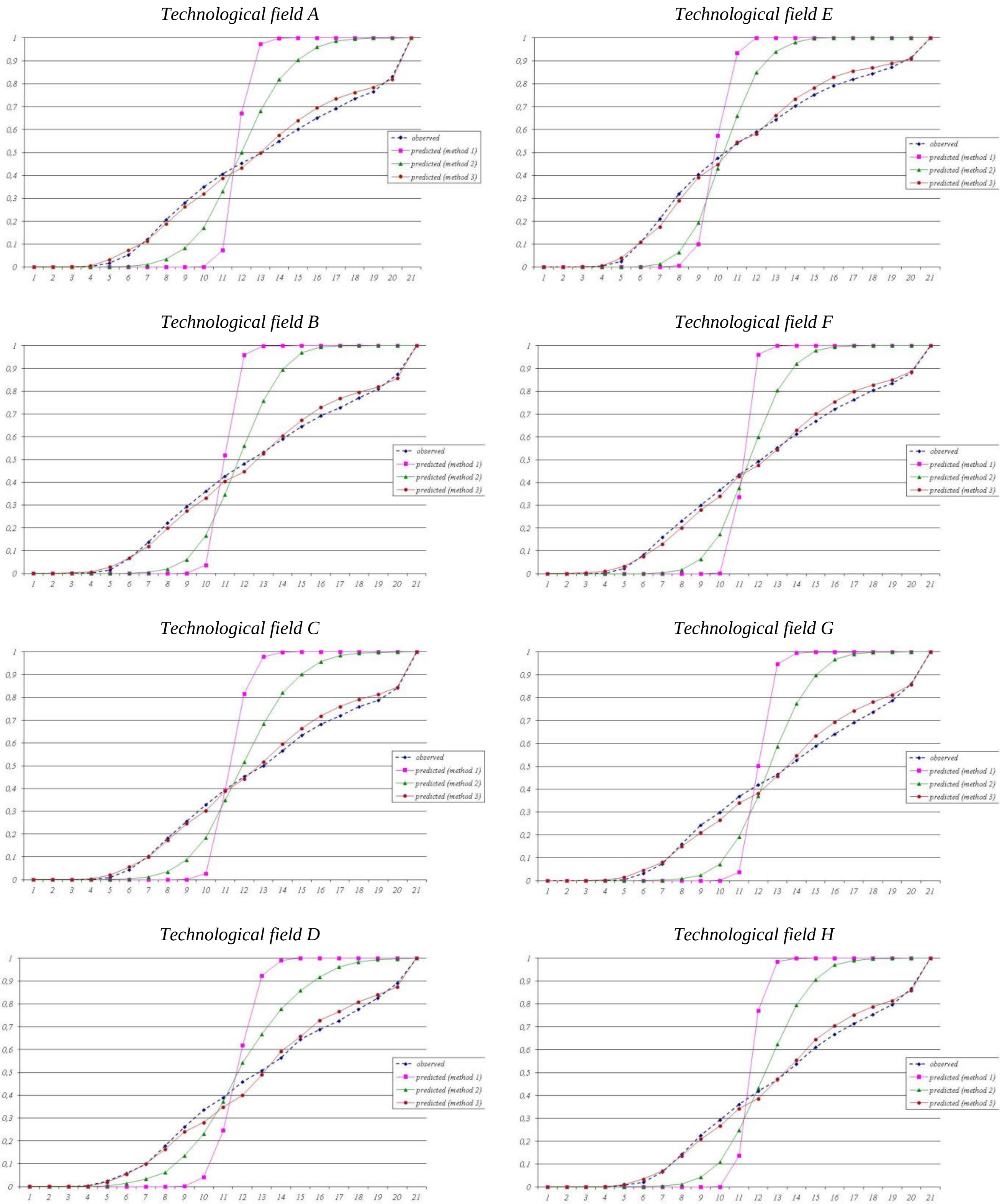

FIGURE 3. OBSERVED AND PREDICTED CUMULATED FREQUENCIES OF WITHDRAWAL

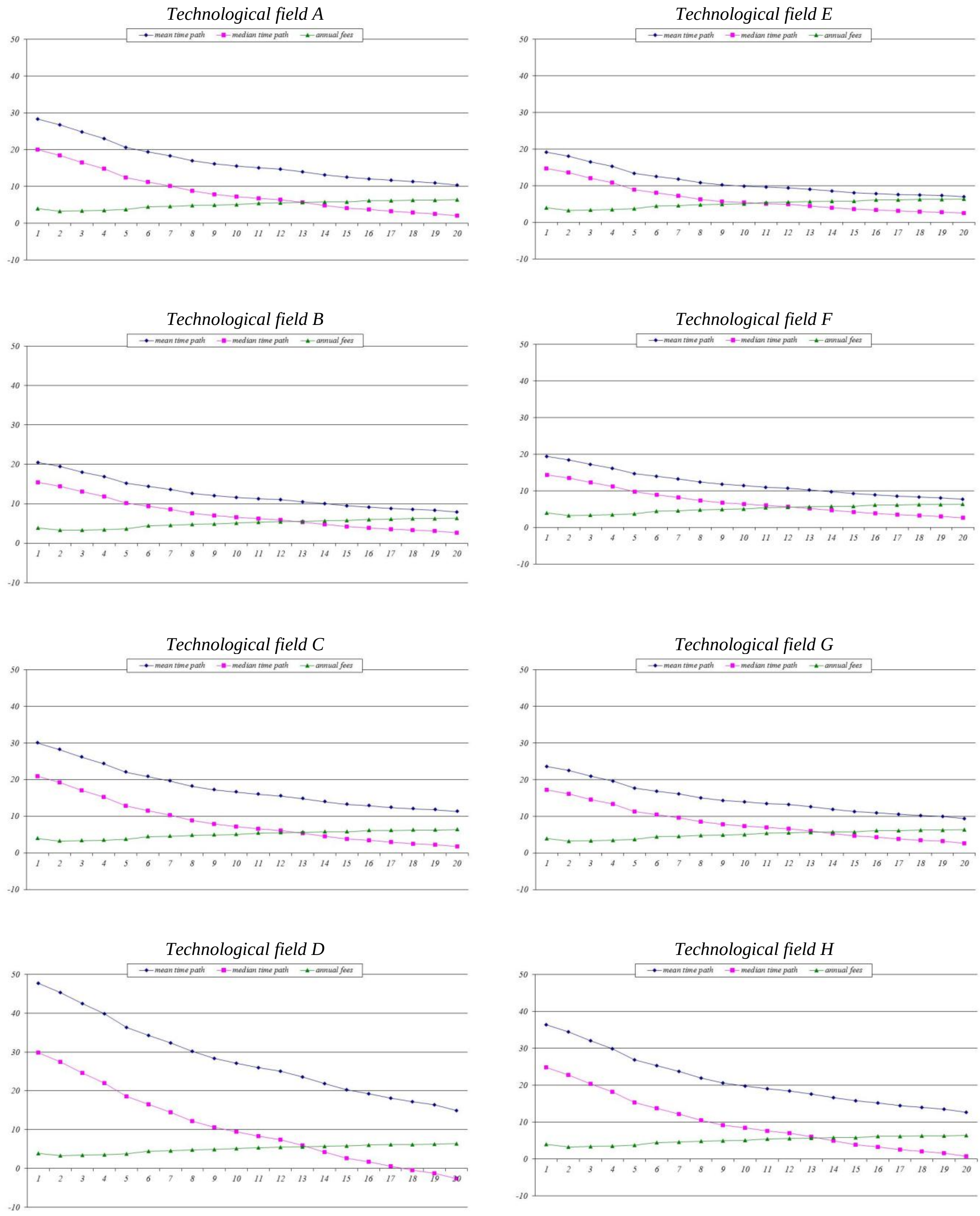

FIGURE 4. RENT DYNAMICS (NATURAL LOGARITHM OF THE RENT AND RENEWAL FEES AS FUNCTIONS OF AGE)

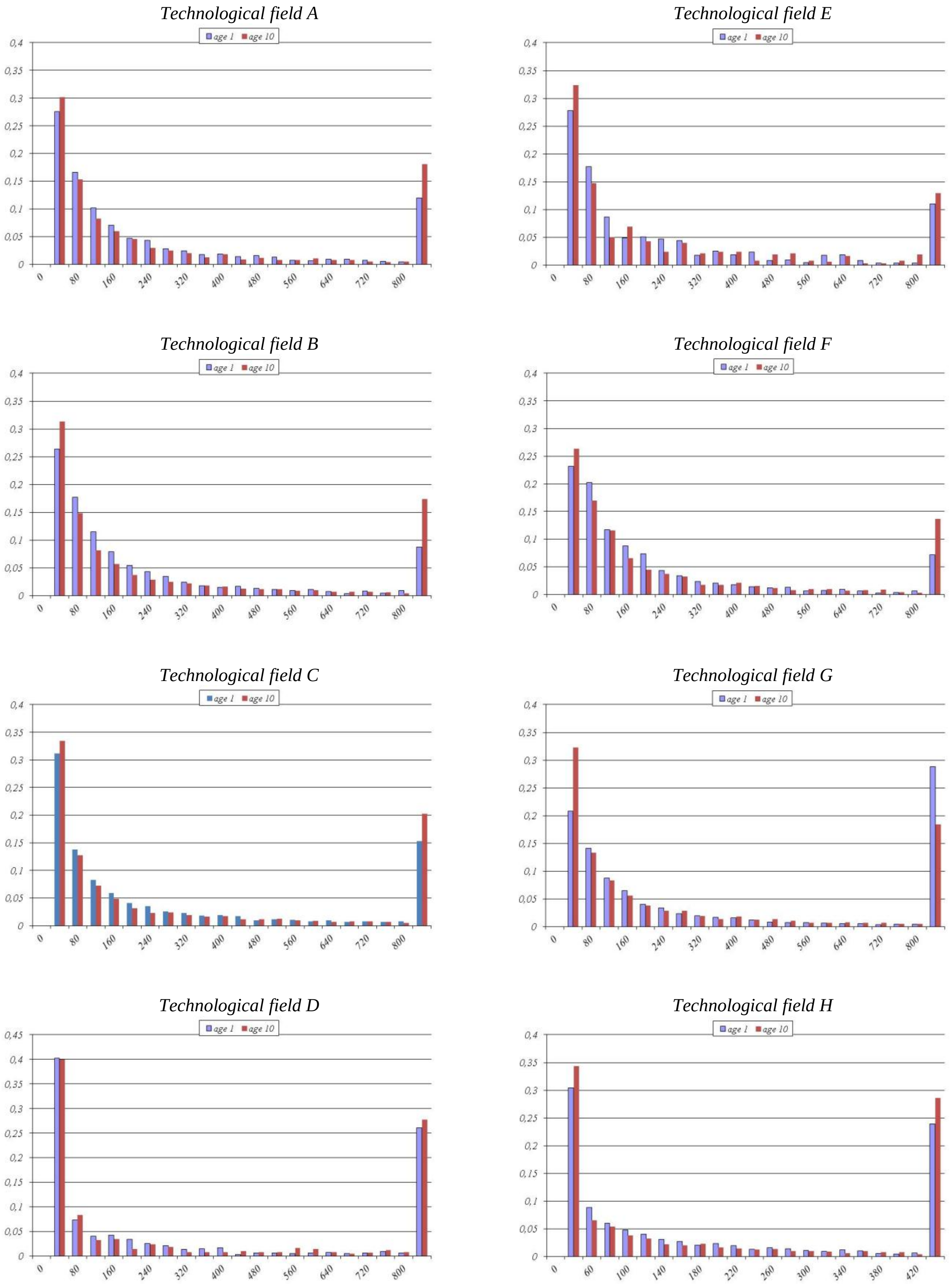

FIGURE 5. EMPIRICAL DISTRIBUTIONS OF IPQS FOR EACH TECHNOLOGICAL FIELD

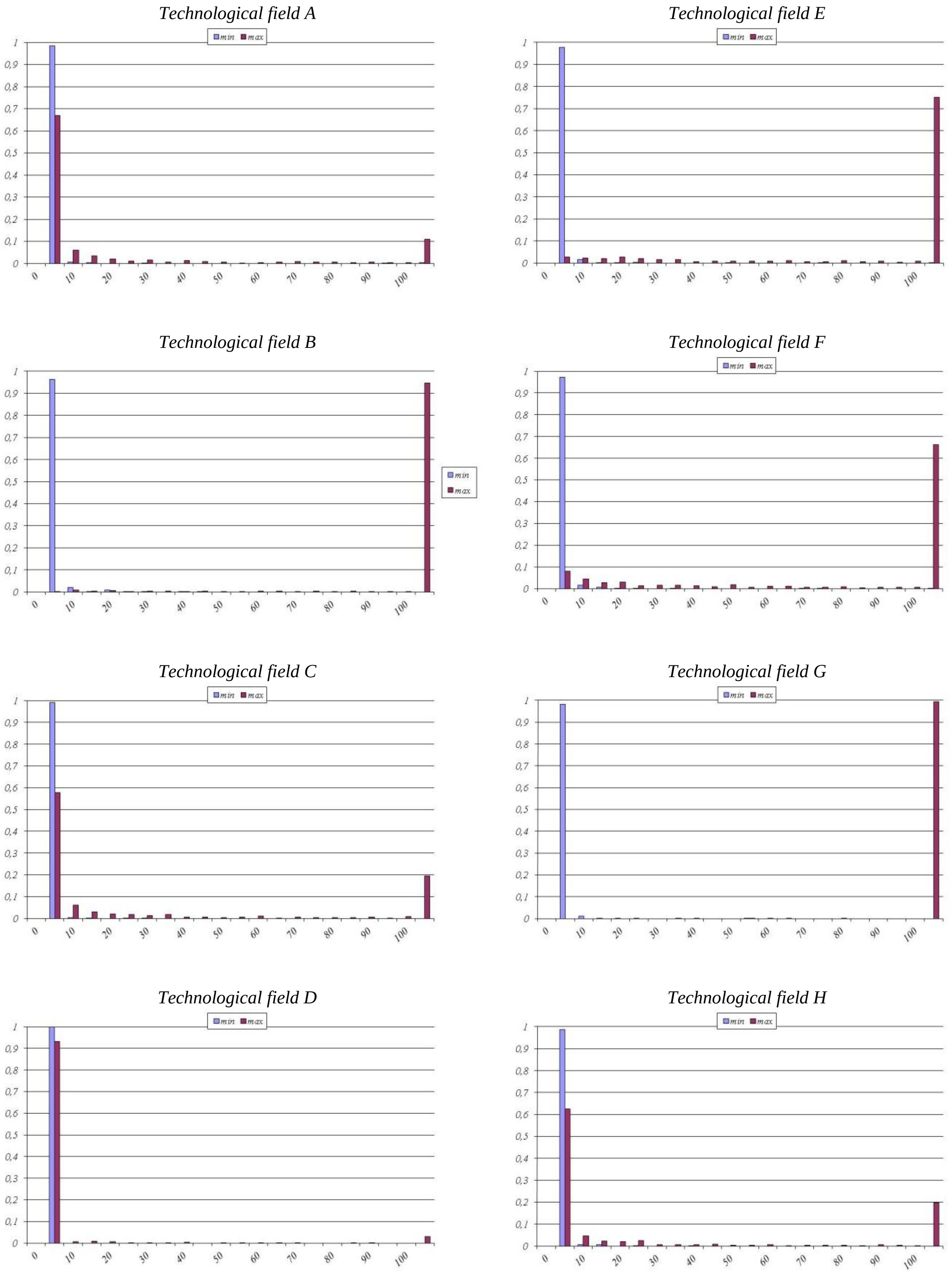

FIGURE 6. DISTRIBUTION OF IPQS OVER ONE THOUSAND RANDOM DRAWS FOR PATENTS WITH THE LOWEST AND THE HIGHEST ESTIMATED IPQ AT AGE ONE

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